Discontinuous constrained-layer damping treatments applied to a vibrating free-free beam

by

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ABSTRACT

The vibration response to application of discontinuous constrained-layer damping (CLD) patches to varied portions of a free-free beam was studied. Since neither closed-form solutions nor finite element methods can be readily implemented for CLD analysis with partial coverage of a structure under broadband excitation, an empirical approach was taken. The test object was a 26.5" x 1" x 1/8" steel beam, and was subjected to a chirp. The aim was to quantify the relative effectiveness of varied lengths and positions of CLD patches in reducing the response of the first five resonant modes of the beam. Assessment of the potential for strain energy dissipation was made based on the net displacement of the beam (appropriately phased mode summation), with relative phase of the summed modes specific to the input force signal. Attempts were made to correlate the effect of CLD on each mode to the time-averaged percentage of strain energy for that mode under the treatment patch, coupled with the percentage of the beam’s strain energy under the patch. [Research supported by NSF]
1. BACKGROUND

1.1 Surface Damping Treatments

1.1.1 Introductory remarks

Surface damping treatments are often used to solve a variety of resonant noise and vibration problems, especially those associated with sheet metal structure vibration. Such treatments can easily be applied to existing structures and provide high damping capability over wide temperature and frequency ranges.

Surface treatments were first studied in the 1950's by Oberst, Kerwin and Ross. Initial attempts at characterizing the behavior of viscoelastic materials used in such treatments were using analytical or closed-form methods using applied mechanics. Theories put forward in the 1960's incorporated many assumptions on the kind of base metal they were applied to, and the kind of boundary conditions they were subjected to. These surface treatments were only subjected to extensional deformation, and they were called Extensional or unconstrained-layer damping treatments.

Around the same time developments were being made in the use of constrained surface treatments, wherein a thin metallic layer was added on top of the viscoelastic layer itself. It was observed that when such a constraint is imposed upon the viscoelastic layer it undergoes shear deformation, thereby dissipating even greater amounts of energy. These treatments were called Shear or constrained-layer damping (CLD) treatments.

Since the 1970's various analytical and finite element theories characterizing the constrained-layer viscoelastic treatments have been put forward. However until the late 1980's the most common use of such treatments was in form of a sandwich structure. The whole surface of the base metal was covered with such treatments. These sandwich structures have met great success in aerospace, automobile and other big industries.

However, lack of a theory regarding discontinuous treatments has hampered the optimized use of constrained-layer treatments. Even more so, most of the theories put forward have concentrated on the excitation of the base structure at a single frequency.
This is rarely, if ever, the case in real world applications of the surface treatments. Even though finite element modeling of such treatments has made progress in working towards discontinuous patches, modeling of such patches under broadband excitation has not been addressed.

1.1.1.1 Purpose of this work

The purpose of this work is to try to characterize the effectiveness of discontinuous surface damping treatments applied to structures under broadband excitation. This will not only optimize the usage of surface patches, but will also address the real world applications of surface treatments. Also of interest is whether discontinuous patches can be applied in such a manner that specific modes of vibration of the structure are affected. This mode-picking property will also be useful in reducing the amount of acoustically radiated sound from vibrating structures, which in many cases is due to specific modes of vibration of the structure.

CLD adds on to the damping in the system. This has the effect of adding another path to the system for energy dissipation. This reduces the vibration amplitudes of the structure to which it is applied. This is shown schematically in Fig. (1.1).

![Schematic diagram for paths of energy dissipation](image_url)
The structure is excited by some sort of excitation causing vibrations in it. Note how the excitation system itself may act as an energy absorber. This is expected of constant voltage excitations, such as a shaker used in vibration testing (see chapter 3 for more information). But the main source of energy dissipation in a bare structure is its inherent damping. When CLD is applied to a structure, it adds damping to the system. This damping, if properly tapped can be several times the inherent damping of the system itself, thereby reducing the vibration amplitudes significantly.

This work starts by taking a look at the theories developed in recent years for characterizing the behavior of CLD treatments. Then the limitations of these theories is presented, and a new direction for predicting vibration reductions using CLD treatments is proposed. A new model for energy dissipation in the CLD is presented, and experiments conducted to test this model are presented, processed and analyzed.

1.1.2 Ross-Kerwin-Ungar analytical analysis of three-layer damped systems

1.1.2.1 Introduction

A variety of approaches have been considered to describe the behavior of different types of surface damping treatments. Of these, the analysis developed by Ross, Ungar, and Kerwin [1] has been the most widely used. This analysis (referred to as the RKU analysis) has been developed for a three-layer system and is usually used to handle both extensional and shear types of treatment. The analysis, within its limitations, can be extended to handle the response of damped plates as well as damped beams.

Although this analysis has been developed to predict the response of damped three-layer systems, assuming that the properties of the damping material are known, it has on numerous occasions been used in reverse. In such cases the damping properties of the material are computed from the damped response of the system, most often a sandwich beam. A discussion of the basic analysis and how it can be extended for different types of treatments and objectives follow.
1.1.2.2 The Ross-Kerwin-Ungar equations

The flexural rigidity, EI, of the three-layer system of Fig. 1.1 is [1]:

\[
EI = E_1 \frac{H_1^3}{12} + E_2 \frac{H_2^3}{12} + E_3 \frac{H_3^3}{12} - E_2 \frac{H_2^2}{12} \left( \frac{H_{31} - D}{1 + g} \right) + E_1 H_1 D^2 + E_2 H_2 (H_{21} - D)^2 + E_3 H_3 (H_{31} - D)^2 \left( \frac{H_{31} - D}{1 + g} \right)
\]  

(1.1)

where

\[
D = \frac{E_2 H_2 (H_{21} - H_{31} / 2) + g (E_2 H_2 H_{21} + E_3 H_3 H_{31})}{E_1 H_1 + E_2 H_2 / 2 + g (E_1 H_1 + E_2 H_2 + E_3 H_3)}
\]  

(1.2)

\[
H_{31} = \frac{(H_1 + H_3)}{2} + H_2
\]  

(1.3)

\[
H_{21} = \frac{(H_1 + H_2)}{2}
\]  

(1.4)

\[
g = \frac{G_2}{E_3 H_3 H_2 p_2}
\]  

(1.5)

E = Young’s modulus of elasticity

G = shear modulus

I = second moment of area

H = thickness

D = distance from the neutral axis of the three layer system to that of the original beam H_1
Subscript 1 refers to the base structure, subscript 2 to the damping layer, subscript 3 to the constraining layer, and no subscript refers to the composite system.

1.1.2.3 Assumptions and Precautions

The major assumptions employed in the RKU analysis were:

1. The most important assumption to keep in mind while using the RKU analysis is that eqs. (1.1) through (1.5) were developed and solved using sinusoidal expansions for the modes of vibration. The beam is assumed to be infinitely long so that the end effects may be neglected. Therefore, the Ross-Ungar-Kerwin analysis applies only to simply supported beams or plates. For other types of boundary conditions, approximations must be used depending on the mode shape of the structure in question.

2. For the entire composite structure cross section, there is a neutral axis, whose location varies with frequency.

3. There is no slipping between the elastic and viscoelastic layers at their interfaces. Thus the analysis assumes rigid connections between various layers of the system. However, in practice, because most damping materials are not self-adhesive, an additional adhesive layer is used for fastening purposes. In such cases the thickness of the adhesive layer must be kept to a minimum, and the modulus of elasticity of its material must be as high as possible. The loss factor of the adhesive should also be low.

4. The major part of the damping is due to the shearing of the viscoelastic material, whose shear modulus is represented by complex quantities in terms of real shear moduli and loss factors.

5. The elastic layers displace laterally by the same amount.

1.1.2.4 RKU analysis for a simply supported beam

To use the RKU analysis to predict the damped response of simply supported beams, it is sufficient to keep in mind that the natural frequency, \( \omega_n \), for such a beam is:

\[
\omega_n = \frac{\zeta}{L} \frac{1}{\sqrt{\frac{EI}{\rho Hb}}} \tag{1.6}
\]
where
\[ \zeta_n = n\pi, \text{ is the } n\text{th eigenvalue} \]
\[ n = \text{mode number} \]
\[ L, b, H = \text{length, breadth and thickness, respectively, of the beam} \]
\[ \rho = \text{mass density} \]

1.1.3 Extensional damping treatment

Extensional damping is the simplest way of introducing damping into a sheet metal type of structure. This treatment is also referred to as the unconstrained-layer or free-layer damping treatment. The treatment consists of a simple layer of damping material bonded to those surfaces of the structure that are vibrating primarily in a bending type of mode. As these surfaces bend, the treatments on the surfaces are deformed cyclically in tension-compression, dissipating energy. This damping treatment is illustrated in Fig. 1.2.

![Diagram of unconstrained or free-layer damping treatment](image)

**Fig. 1.2 Unconstrained or free-layer damping treatment**

Implicit in this method is that the shear deformations can be ignored and simple beam or plate bending theory for layered structures can be used to analyze these types of damping treatments. The deformation of the viscoelastic surface treatment follows that of the base structure because of the large difference in flexural rigidity of the base structure
and the viscoelastic material. This type of surface treatment will, in general, not enhance vibration damping of the structure effectively.

The Ross-Kerwin-Ungar (RKU) equations can be applied to predict the performance of unconstrained-layer damping treatments. To use this analysis, it is necessary to consider the special case for which the constrained-layer thickness, $H_3$, is zero. This will reduce the earlier analysis of the three-layer system to a two-layer system. As $H_3$ approaches zero, the shear parameter, $\gamma$, approaches infinity, and eq. (1.1) can be simplified to:

$$\frac{EI}{E_1 I_1} = 1 + e_2 h_2^3 + 3(1 + h_2)^2 \left( \frac{e_2 h_2}{1 + e_2 h_2} \right)$$  \hspace{1cm} (1.7)

where

$$e_2 = \frac{E_2}{E_1}$$
$$h_2 = \frac{H_2}{H_1}$$

If the loss factor of the damping material is $\eta_2$, and the system loss factor is $\eta$, eq. (1.7) can be rewritten as:

$$\frac{EI}{E_1 I_1} (1 + i\eta) = 1 + e_2 h_2^3 (1 + i\eta_2) + 3(1 + h_2)^2 \left( \frac{e_2 h_2 (1 + i\eta_2)}{1 + e_2 h_2 (1 + i\eta_2)} \right)$$  \hspace{1cm} (1.8)

Equating the real and imaginary parts on both sides of eq. (1.8), and noting that for practical situations $(e_2 h_2)^2 << e_2 h_2$, the following two equations can be derived:

$$\frac{EI}{E_1 I_1} = \frac{1 + 4e_2 h_2 + 6e_2 h_2^3 + 4e_2 h_2^3 + e_2^2 h_2^4}{1 + e_2 h_2}$$  \hspace{1cm} (1.9)

$$\frac{\eta}{\eta_2} = \frac{e_2 h_2 (3 + 6h_2 + 4h_2^2 + 2e_2 h_2^2 + e_2^2 h_2^4)}{(1 + e_2 h_2) (1 + 4e_2 h_2 + 6e_2 h_2^2 + 4e_2 h_2^3 + e_2^2 h_2^4)}$$  \hspace{1cm} (1.10)
It is interesting to note that eqs. (1.9) and (1.10), for predicting the damping performance of free-layer treatments, are the same as those reported by Oberst[2]. Eq. (1.9) is used to compute the effective system stiffness for use in frequency prediction, whereas eq. (1.10) is used to predict the damping performance.

For a further discussion of extensional damping treatments, such as effects of temperature, thickness, frequency, semiwavelengths, bonding techniques, etc., see [3].

1.1.4 Shear damping treatment

Shear damping treatment is similar to the unconstrained-layer type, except that the free surface of the viscoelastic material is constrained by a metal layer. The flexural modulus of the constraining layer is comparable to that of the base structure. When the structure is subjected to cyclic bending, the metal layer will constrain the viscoelastic material and force it to deform in shear. This occurs due to the excessive difference of moduli between the viscoelastic material, the base structure, and the constraining layer. Shear is the mechanism by which energy is dissipated. This concept is illustrated in Fig. 1.3.

![Diagram of shear damping treatment](image)

**Figure 1.3 Single layer constrained-layer treatment**
To illustrate the mechanism for energy dissipation further, consider the two extremes of the middle layer damping properties for Fig. 1.3. At low temperatures, where the material is in its glassy region, both the structure and the constrained layer become rigidly coupled. In this case, whenever the system is subjected to cyclic bending, little shear deformation occurs in the middle layer, hence the energy dissipation is also small. On the other hand, at high temperature, where the viscoelastic material is in its rubbery region and soft, both the structure and constrained layer become almost uncoupled. The energy dissipation in this case is also minimal, even though the shear deformation in the middle layer is high. This is because the shear modulus of the middle layer is low. Between these two extremes, the material possesses an optimal modulus value, so that the energy dissipation for the constrained layer goes through a maximum. The maximum shear deformation in the middle layer is a function of the modulus and the thickness of the constraining layer, the thickness of the damping layer, and the wavelength of vibration in addition to the properties of the damping material. The term that contains these variables is the shear parameter $g$, given in eq. (1.5).

The performance of the constrained-layer damping treatment depends to a large extent on the geometry and type of constraining layer. Usually, it is desirable to have the constraining layer as stiff as possible to introduce the maximum shear strains into the viscoelastic layer. However, the constraining-layer stiffness should not normally exceed that of the structure. Therefore, the maximum amount of shear strain is usually accomplished whenever the constraining-layer is of the same type and geometry as that of the structure to be damped. This is usually referred to as the sandwich damping treatment and is illustrated in Fig. 1.4.

Multiple constrained-layer treatments are usually used as a means for increasing the damping introduced into a structure. Increasing the number of layers has the same effect as increasing the thickness of the constrained layer, but with slight variations. Fig. 1.5 illustrates two constrained-layer treatments on a structure. The specific performance of such treatments can be found in [3], but it should be noted that the shear strain introduced into the second viscoelastic layer is less than that of the first layer.
The shear type of damping treatment is more efficient than the unconstrained-layer damping treatment, for a given weight. However, this efficiency is balanced by greater complication in analysis and application. This was evident from the RKU analysis presented in section 1.1.2, and will further be exemplified in other historical analyses as presented in the next section.
1.2 Analytical Vibrational Analyses of Three-Layered Damped Beam Structures

1.2.1 Introduction

Representing the simplest constrained damped structure, the three-layer damped laminate has long been of interest to many investigators. The laminate is composed of a viscoelastic damping layer sandwiched between two elastic face layers. As explained in section 1.1, when the beam or plate undergoes flexural vibration, the damped core is constrained to shear.

Because of the importance in understanding the fundamental physics and design applications of damped sandwich structures, the development of many interesting theories and the study of vibratory characteristics of such composite structures have received much attention over the years. The fundamental work in this field was done by Ross, Ungar and Kerwin (RKU analysis). Section 1.1.2.2 presented the RKU analysis in condensed format. This section outlines the vibration analyses developed by other investigators with appropriate references for the more involved reader.

1.2.2 Vibration analysis developed by DiTaranto [4]

In extending the work of Ross, Kerwin and Ungar, DiTaranto eliminated the simply supported end condition assumption and derived an equation of motion for freely vibrating beams having any end conditions. The equation of motion is a six-order linear homogeneous differential equation in terms of longitudinal displacement of a face plate [4].

It has been proved that the natural frequencies and modes discussed by DiTaranto constitute a special class of resonance frequencies and forced modes of vibration of the sandwich beam. This greatly simplifies the general forced vibration problem. However, the modes can only exist in the presence of an external transverse loading that is proportional at all points along the beam to the local inertia loading. At the resonance frequency, the external loading is proportional to the modal loss factor and the inertial loading. This was discussed in detail by Mead and Markus [5].
1.2.3 Vibration analysis developed by Yan and Dowell [6]

Using the principle of virtual work in the theory of elasticity and the theorem of correspondence, a set of five partial differential equations was derived for the governing equations for vibrating three-layered constrained damping sandwich plates and beams. Three branches of dispersion curves were developed associated with these sets of equations. One of the purposes of this work was to obtain a simplified equation that closely describes the actual physical mechanisms, such as the classical Euler beam equation does for simple beams. A simple differential equation for nonsymmetric sandwich plates is deduced. The natural boundary conditions associated with this equation were derived. Typical numerical results were presented including a comparison with experiment.

1.2.4 Vibration analysis developed by Rao and Nakra [7]

Rao and Nakra were one of the first to include the inertial effects of transverse, longitudinal, and rotatory motions for unsymmetrical sandwich structures with viscoelastic cores. The detailed derivation for the equations of motion for the flexural vibration of the unsymmetrical sandwich damped beams can be found in the reference cited.

It was shown that in case of unsymmetrical sandwich beams with elastic cores, inclusion of all the inertia effects in the analysis of flexural vibrations gives three families of modes of vibration for each modal number. These are predominantly flexural, extensional, and thickness shear types of modes respectively. For a simply supported damped beam due to forced harmonic excitation, the influence of inclusion of inertia effects on the response and the variation of system loss factors with various geometrical and material parameters are presented by Rao and Nakra.

1.3 Finite Element Analysis of Damping in Structures with Constrained Viscoelastic Layers

1.3.1 Introduction

In the past decade or so, the finite element analysis has played an important role in the determination of the vibration solutions of structures. Because of the versatility of the
method and the availability of various general purpose computer codes, finite element methods have been used in a large number of practical applications. Much of the difficulty in the analysis and design of layered dampers stems from complicated geometries. It is then natural to look to finite element methods for solutions, just as they are used for the analysis of general undamped structures. In this section, several approaches to damped structural design are reviewed in the context of implementation by existing general-purpose finite element codes.

1.3.2 Johnson and Kienholz's finite element analysis

Johnson and Kienholz [8] have determined the numerical solutions of modal loss factors and natural frequencies of a sandwich cantilever beam for a number of constrained damped structural elements. Their technique of analysis is presented here in brief. There are three stages in arriving at the response predictions by finite element analysis of an integrally damped structure. These are outlined below.

1.3.2.1 Arriving at the desired form of the discretized equations of motion

Three approaches have been used effectively to arrive at the desired mathematical model. Each approach has its inherent advantages and disadvantages.

a) The complex eigenvalue method. Suppose the discretized equations of motion take the form

$$Mx'' + Cx' + Kx = f(t)$$  \hspace{1cm} (1.11)

where

- $M$, $C$, $K =$ physical coordinate mass, damping, and stiffness matrices (all real and constant)
- $x$, $x'$, $x'' =$ vectors of nodal displacements, velocities, and accelerations
- $f(t) =$ vector of applied node loads, also known as the force vector. It reflects the forced excitation introduced in the system to induce vibration.
The solution can be carried out in terms of damped normal modes [9, 10]. Both
the eigenvalues and eigenvectors will in general be complex, but the method is nonetheless
quite standard in that the modes obey an orthogonality condition and thus allow
uncoupled equations of motion to be obtained.

There are two important drawbacks to this method. It is computationally
expensive, typically three times the cost of the corresponding undamped eigensolution
[11]. Also, for a structure to be described by eq. (1.11), its materials, including any
viscoelastic materials, must exhibit dynamic stress-strain behavior of a certain type.
Storage moduli must be constant and loss moduli must increase linearly with frequency
[12]. Real viscoelastic materials simply do not behave in such an accommodating way.
Storage moduli tend to increase monotonically with frequency whereas loss factors exhibit
a single, mild peak [13].

Another finite element procedure that is often referred to as a complex eigenvalue
method is to suppress the $C\mathbf{x}'$ term in eq. (1.11) and treat the stiffness matrix $\mathbf{K}$ as
complex.

b) Direct frequency response method. If the applied load varies sinusoidally in time,
energy dissipation in the structure can be accounted for by treating the elastic constants of
any or all the materials as complex quantities that are functions of frequency and
temperature. These material properties are presumably available from sinusoidal tests. If
the structure is linear, its response will be sinusoidal at the driving frequency, and the
steady-state equations of motion will have the form

$$
\begin{bmatrix}
-M\omega^2 + K_1(\omega) + iK_2(\omega)
\end{bmatrix}
\begin{bmatrix}
\mathbf{X}(\omega)
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{f}(\omega)
\end{bmatrix}
$$

(1.12)

where

$K_1(\omega), K_2(\omega)$ = stiffness matrices calculated using the real and imaginary parts of
material properties, respectively

$\omega$ = radian frequency of excitation

$\mathbf{f}(\omega), \mathbf{X}(\omega)$ = complex amplitude vectors of applied node loads and responses,
respectively
i = \sqrt{-1}

It should be clear that material constants are truly complex quantities only in the sense that complex arithmetic is used simply as a convenient method of keeping track of relative phases under sinusoidal excitation.

There are several drawbacks to this method. It is computationally expensive because a general sinusoidal solution requires that the displacement impedance matrix (the bracketed quantity in eq. (1.12)) be recalculated, decomposed, and stored at each of many frequencies. Further, the method does not give information of direct use to a designer in improving performance of a candidate structure.

The costliness of the direct frequency response method indicated by eq. (1.12) is a direct result of the restriction to physical coordinates (as opposed to modal coordinates). This restriction is, in turn, caused by the form of the corresponding time domain representation. General convolution integral relations between forces and displacements must be admitted in order to accommodate the variation of \( K_1 \) and \( K_2 \) with frequency that is observed in real viscoelastic materials. Because not even the form, let alone the parameter values of the convolution relation are generally known, it must be represented numerically in the frequency domain in terms of its Fourier transform. A tabular frequency representation can be arbitrarily accurate as long as the underlying stress-strain operator is linear. Use of such a data format is bound to be costly, however, particularly if a high level of frequency resolution is required.

c) Modal strain energy method. In this approach it is assumed that the damped structure can be represented in terms of the real normal modes of the associated undamped system if appropriate damping terms are inserted into the uncoupled modal equations of motion. That is,

\[
\alpha''_r + \eta^{(r)} \omega_r \alpha'_r + \omega_r^2 \alpha_r = f_r(t) \quad (1.13)
\]
\[
x = \sum \phi^{(r)} \alpha_r(t) \quad r = 1, 2, 3, \ldots \quad (1.14)
\]
where
\[ \alpha_r = \text{rth modal coordinate} \]
\[ \omega_r = \text{natural radian frequency of the rth mode} \]
\[ \phi_r^{(r)} = \text{rth mode shape vector of the associated undamped system} \]
\[ \eta_r^{(r)} = \text{loss factor of the rth mode} \]

It is implied that the physical coordinate damping matrix \( C \) of eq. (1.11) need not be explicitly calculated but that it can be diagonalized, at least approximately, by the same real modal matrix that diagonalizes \( K \) and \( M \).

The modal loss factors are calculated by using the undamped mode shapes and the material loss factor for each material. This general approach was first suggested by Kerwin and Ungar [14] in 1962. Its application by finite element methods was suggested by Rogers [15] and Medaglia [16]. For sandwich structures, the material loss factor of the metal face sheets is very small compared with that of the viscoelastic core. In this situation, the modal loss factor is found from

\[ \eta_r^{(r)} = \eta_v \left[ \frac{V_r^{(r)}}{V^{(r)}} \right] \]  \hspace{1cm} \text{(1.15)}

where
\[ \eta_v = \text{material loss factor of viscoelastic core evaluated at the rth calculated resonant frequency} \]
\[ V_r^{(r)} / V^{(r)} = \text{fraction of elastic strain energy attributable to the sandwich core when the structure deforms in the rth mode shape} \]

A derivation of eq. (1.15) is given in [8]. It is shown that modal loss factors obtained from eq. (1.15) can be expected to approximate the computationally more expensive complex stiffness eigenvalue results.

Calculation of the modal energy distributions fits quite naturally within finite element methods and is a standard option in some commercial codes [17]. The basic advantages of the method are that only undamped normal modes need be calculated and
that the energy distributions are of direct use to the designer in deciding where to locate damping layers. The disadvantage is that some approximation is required to accommodate frequency-dependent material properties.

1.3.2.2 Choice of elements for modeling the sandwich structure

Regardless of the solution method to be employed, modeling of sandwich structures requires that the strain energy due to shearing of the core be accurately represented. Practical considerations dictate that this be done with minimum increase in computation cost relative to a uniform, single-layer model. In this section, two effective modeling methods are described.

a) Elements used by Johnson and Kienholz [8]. These researchers use a modeling method that is reasonably efficient and has the important advantage of being readily implemented in MSC/NASTRAN®, a widely available code. Fig. (1.6) shows the arrangement for modeling of a three-layer sandwich.

The face sheets are modeled with quadrilateral or triangular plate elements producing stiffness at two rotational and three translational degrees of freedom per node. The viscoelastic core is modeled with solid elements producing stiffness at three translational degrees of freedom per node. All nodes are at element corners. The plate elements are called TRIA3, QUAD4, TRIA6, and QUAD8, and the solid elements are called PENTA and HEXA in MSC/NASTRAN®. A key feature of these plate elements in this modeling is their ability to account for coupling between stretching and bending deformations [18]. This feature allows the plate nodes to be offset to one surface of the plate, coincident with the corner nodes of the adjoining solid elements. In this way, a three-layer plate can be modeled with only two layers of nodes. Aspect ratios of the solid elements (in-plane dimension to thickness dimension) as high as 5000 can be used to model the viscoelastic core layers.
b) Elements used for modeling by Rao [19]. According to Rao (and as reflected in the previous section also), for beams with a constrained viscoelastic layer and a constraining layer, it is very convenient to use the offset beam element in the finite element formulation. A typical finite element mesh used for his modeling of the three-layer sandwich beams is shown in Fig. 1.7 and 1.8.

Rao’s model differs from that of Johnson and Kienholz’s in the degrees of freedom provided to the individual elements. The base structure and the constraining layer were modeled by using a specially developed three-node shear deformable, seven-degree-of-freedom, offset beam element Fig.1.8(a). The term shear deformable implies that the kinematic assumptions are based on the Timoshenko beam theory, which is significant for fiber-reinforced composite base structures. The viscoelastic core is modeled by using a rectangular plane element with the offset element Fig. 1.8(b).

In order to be compatible with the beam element, the displacement $u$ in the $x$-direction is interpolated by using linear functions. The nodal displacements $w$ in the $z$-direction are interpolated using quadratic interpolation functions in the $x$-direction and linear interpolation functions in the $z$-direction. The use of higher order interpolation functions in the $x$-direction for $w$ improves the performance of the element in situations where the loading causes $w$ to be much larger than $u$, as in beam bending problems.
1.3.2.3 Solution of the equations of motion of the sandwich structure

Once the mathematical model is assembled, either the frequency response or modal strain energy analysis can be performed. In the direct frequency response analysis, the core material properties are input via a table as complex functions of frequency, and the solution proceeds as described earlier.

In the modal strain energy method, a standard normal mode extraction run is made with all material constants treated as real and constant. The elastic strain energy in each
element for each mode is calculated as well as the energy fraction in the viscoelastic core for each mode. These fractions multiplied by the core material loss factor give the modal loss factors, which are input via a damping-vs-frequency table for use in subsequent forced response calculations.

1.3.3 Limitations of the finite element analysis of sandwich structures

The modal strain energy method of solution is the most widely used and the most effective method for finite element analysis [8]. However it suffers from some serious drawbacks.

A basic difficulty with the modal strain energy method (or any other normal mode method) is that the modal properties are obtained from system matrices that are assumed to be constant. Viscoelastic materials, however, have storage moduli that vary significantly with frequency. There is no theoretically correct way to resolve this contradiction. There, are, however, great practical advantages to making response predictions in terms of a normal mode set obtained from constant material properties. This can be done with reasonable accuracy if a simple correction is made to the modal loss factors obtained by eq. (1.15). This correction is only to the modal parameters that can be readily adjusted by the finite element analyst.

For broadband excitation, most of the response of a given mode occurs within a narrow band around the mode's natural frequency. It is natural, then, to require that the energy distribution used to compute the loss factor for a given mode be obtained using a stiffness matrix evaluated for material properties taken at that mode's frequency. Because the natural frequencies themselves depend on material properties, an iterative solution of two simultaneous relations (the eigenvalue problem for each mode number and the material property vs frequency relation) is required. This is readily done [15], but a further problem remains. The final modal coordinate representation of the structure must come from a single stiffness matrix evaluated using a single value of storage modulus for the core material. Natural frequencies, mode shapes, and modal masses will be correct
for, at most, one mode. A further empirical correction of the modal loss factor has been found to give some improvement. Details of such correction can be found in [8].

1.3.4 Direction for current work

In the previous section, the various limitations of the finite element method were illustrated. The biggest limitation is the lack of analysis of structures under broadband excitation. Also missing in such a scenario is the application of discontinuous patches of treatments, which would optimize its usage.

As noted in the introduction of this chapter, both the above are true in a real world scenario of application of constrained-layer damping (CLD) treatments. We would also like to be able to reduce the effect of a particularly troublesome mode of frequency of vibration, or even of noise radiation. But at the same time we would also like to optimize the usage of surface damping material.

If we look back at the way CLD works, we will see that it is due to dissipation of energy due to shear deformation. This dissipated energy provides extra damping to the system, thereby reducing the magnitudes of vibration. Therefore the path of characterization of CLD should lie alongside the development of a model for the energy contained in the structure during vibration. This energy would be nothing else but the strain energy associated with each mode of vibration.

1.3.5 Choosing the correct structure for testing a new model

The long term goal for developing a new working model would be one that is able to predict the response of the structure depending on how much and where the CLD patches are applied. While in real world applications the structures we encounter are very complex, such structures are not suitable for testing new models or theories. This is because when we don’t exactly know how the structure itself behaves (lack of an easy or perfect analytical model for its behavior), it is very hard to test for changes corresponding to any applied treatment.

For this reason we turn to the simplest form of structure available, a beam i.e. a
one-dimensional structure. Even more we chose a free-free beam to work with for reasons presented in chapter 3 of this work. It is sufficient to say for now that a free-free beam provides very replicable results, something essential for testing any sort of changes in amplitude of vibration.

We now turn towards developing a model for strain energy variation along the length of a free-free beam to work along the path of a more generalized model for a more generalized structure. Such a model would definitely help in characterizing the effectiveness of CLD in reducing system response for one-dimensional structures. If this model can be shown to work for such a simple structure, then there is definite hope that it would work for more complex models.
2. STRAIN ENERGY FORMULATIONS

2.1 Strain Energy Concepts

2.1.1 Introduction

In mechanics, energy is defined as the capacity to do work, and work is the product of force by the distance in the direction the force moves. In solid deformable bodies, stresses multiplied by their respective areas are forces, and deformations are distances. The product of these two quantities is the internal work done in a body by externally applied forces. This internal work is stored in a body as the internal elastic energy of deformation, or the elastic strain energy. In this section, the strain energy equation for solid materials will be presented in condensed format.

2.1.2 Elastic strain energy for isotropic, linear, and elastic materials

Consider a solid maintained in equilibrium in space by system of supporting forces, in Fig. 2.1. A system of external surface tractions and body forces is applied in a quasi-static manner. This means that forces are applied so slowly that dynamic effects, such as vibrations, can be neglected.

Figure 2.1 Body in equilibrium
The first law of thermodynamics requires that

\[ Q - W_k = E_2 - E_1 \]  \hspace{1cm} (2.1)

where

\begin{align*}
Q & \quad = \text{heat transfer from surroundings to the body during the loading process} \\
-W_k & \quad = \text{work done by the external forces during the loading process} \\
E_2 - E_1 & \quad = \text{change of stored energy of the body from heating and loading}
\end{align*}

It should be clear that the constraining forces do no work since they do not move. If the process is close to being adiabatic (no heat transfer), then we can drop the term \( Q \). Also, if the work of gravity is negligible during the process (i.e., the center of gravity does not appreciably change position during the loading process), then the change in stored energy, \( E_2 - E_1 \), is merely the change in internal energy of the material resulting from the work of the surface tractions and body forces excluding gravity. Now if the material is elastic (not necessarily linear, elastic), there is no hysteresis. Then the body will perform an equal but opposite amount of work on the surroundings during the unloading process, as was done by the surroundings on it during the loading process.

Thus, we can consider for such bodies that energy has been stored in the body as a result of the deformation, and this stored energy equals the external work done on the body. This stored energy, which is available as work on releasing the applied loads in a quasistatic manner, in elastic solids is called strain energy of equilibrium.

The formulation of computing the strain energy directly from the stress-strain distribution in the body with can be found in several solid mechanics texts [20(a), 21]. The strain energy per unit volume, \( \varepsilon \), can be given in terms of stresses as:

\[ \varepsilon = \frac{1}{2E} (\tau_{xx} + \tau_{yy} + \tau_{zz}) - \frac{\nu}{E} (\tau_{xx} \tau_{yy} + \tau_{xx} \tau_{zz} + \tau_{yy} \tau_{zz}) + \frac{1}{2G} (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \]  \hspace{1cm} (2.2)

where
\( E = \) modulus of elasticity

\( \nu = \) Poisson’s ratio

\( G = \) modulus of rigidity

\( \tau_{ii} = \) normal stress, and \( i = x, y, z \)

\( \tau_{ij} = \) shear stress, and \( i, j = x, y, z \) \( (i \neq j) \)

The strain energy per unit volume at a point can also be given in terms of the strains [20(a), 21] as:

\[
\mathcal{U} = \frac{1}{2} \frac{E \nu}{(1 + \nu) (1 - 2\nu)} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})^2 + G (\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2)
\]

\[
+ \frac{1}{2} G (\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2)
\]  

(2.3)

where

\( \varepsilon_{ii} = \) normal strain, and \( i = x, y, z \)

\( \gamma_{ij} = \) shear strain, and \( i, j = x, y, z \) \( (i \neq j) \)

The total strain energy, \( U \), for a given body of volume \( V \) can then be given in the following ways [20(a), 21]:

\[
U = \int \left[ \frac{1}{2E} (\tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2) - \frac{\nu}{E} (\tau_{xx}\tau_{yy} + \tau_{xx}\tau_{zz} + \tau_{yy}\tau_{zz}) + \frac{1}{2G} (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right] \, dV
\]

(2.4)

\[
U = \int \left[ \frac{1}{2} \frac{E \nu}{(1 + \nu) (1 - 2\nu)} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})^2 + G (\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2)
\]

\[
+ \frac{1}{2} G (\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2) \right] \, dV
\]  

(2.5)
2.1.3 Strain energy for pure bending of symmetric beams

2.1.3.1 Pure bending of linear, elastic, symmetric beams

Let us consider a weightless beam loaded by couples of magnitude \( M_z \) at the ends as shown in Fig. 2.2, where the undeformed geometry of the beam is drawn. The cross-section of the beam is symmetric about an axis that we have chosen as the y axis. The x axis has been shown running parallel to the centerline of the beam. The z axis forms a right-handed triad with our x and y axis.

![Figure 2.2 Pure bending](image)

For long, slender beams undergoing small deformations, the Euler-Bernoulli theory of bending [20(b)] states that at a point on the beam:

\[
M_z = \frac{E I_{zz}}{R} \tag{2.7}
\]

\[
\tau_{xx} = -\frac{M_z y}{I_{zz}} = -\frac{E y}{R} \tag{2.8}
\]

\[
\tau_{yy} = \tau_{zz} = \tau_{xy} = \tau_{yz} = \tau_{xz} = 0 \tag{2.9}
\]

where

\( M_z \) = couple (or moment) about the z-axis

\( E \) = modulus of elasticity

\( R \) = radius of curvature for the beam
\[ y = \text{distance from the neutral axis to the point in consideration} \]
\[ I_{zz} = \text{second moment of area of the cross-sectional area about the neutral axis} \]
(neutral axis is the intersection of the neutral surface and cross section in undeformed geometry)

### 2.1.3.2 Strain energy formulation

As noted from eqs. (2.8) and (2.9), \( \tau_{xx} \) is the only non-zero stress in a beam subjected to pure bending (Fig. 2.3), with an arbitrary cross-section. We can substitute the value for \( \tau_{xx} \) from eq. (2.8) into eq. (2.3) to obtain the expression for the strain energy at a point in a beam under bending. The strain energy at a point for pure bending is then, using eq. (2.3):

\[
\mathcal{U} = \frac{1}{2E} \tau_{xx}^2
\]

(2.10)

Substituting eq. (2.8) above we get

\[
\mathcal{U} = \frac{E y^2}{2} \left( \frac{1}{R^2} \right)
\]

(2.11)

![Figure 2.3 Cross section of a beam showing stress due to bending](image-url)
We can express the radius of curvature $R$ in terms of the deflection curve (see Fig. 2.4) by the following well-known formulation from analytic geometry [20(c)]:

$$
\frac{1}{R} = \frac{d^2 Y}{dx^2} \left[ 1 + \left(\frac{d Y}{dx}\right)^2 \right]^{3/2}
$$

where

\[ Y = Y(x) \]

is the deflection of the neutral surface, positive pointing upward.

Since we are restricted to small deformations by the Euler beam theory, the slope $dY/dx$ will be small, so that the term $(dY/dx)^2$ can be neglected when compared to unity. The foregoing relation then becomes:

$$
\frac{1}{R} = \frac{d^2 Y}{dx^2}
$$

Substituting for $R$ in eq. (2.11) using the preceding relation gives us:

$$
\mathcal{U} = \frac{E y^2}{2} \left(\frac{d^2 Y}{dx^2}\right)^2
$$

Note that $\mathcal{U} = F(x, y)$, where $F$ is some arbitrary function. Since $E$ is a constant, and so is $y$ for a particular point, we have:
\[ \psi = F \left( \frac{d^2 Y}{dx^2} \right)^2 \]  
\hspace{1cm} (2.15)

2.2 Instantaneous Strain Energy for a Vibrating Free-Free Beam

2.2.1 Introduction

It is evident from eq. (2.15) that to determine the strain energy in a beam deformed under pure bending, we only need to determine the deflection of the beam, \( Y(x) \). We shall now examine how to determine the strain energy associated with a vibrating free-free beam. It is to be noted that the "deflection" of the free-free beam under excitation will just pertain to the sum of the modes of vibration.

2.2.2 Formulation

The total deflection of a free-free beam is:

\[ Y(x) = \sum_{i=1}^{n} Y_i(x) \]  
\hspace{1cm} (2.16)

where \( Y_i(x) \) is the deflection for the \( i \)th mode of vibration. The analytical formulation for modes of a free-free beam can be found in any elementary book on vibrations such as [22], and is quoted here:

\[ Y_i(x) = \cosh (\beta_i x) + \cos (\beta_i x) - \sigma_i \left[ \sinh (\beta_i x) + \sin (\beta_i x) \right] \]  
\hspace{1cm} (2.17)

where

\[ \sigma_i = \frac{\cosh (\beta_i l) - \cos (\beta_i l)}{\sinh (\beta_i l) - \sin (\beta_i l)} \]  
\hspace{1cm} (2.18)

\[ \beta_i = \text{solutions of the equation: } \cos (\beta l) \cosh (\beta l) = 1 \]

\[ l = \text{length of the beam} \]

Taking the second derivative of the modal displacements in eq. (2.17) with respect to \( x \), we obtain:
\[
\frac{d^2 Y_i(x)}{dx^2} = \beta_i^2 \{ \cosh (\beta_i x) - \cos (\beta_i x) - \sigma_i [\sinh (\beta_i x) - \sin (\beta_i x)] \} \tag{2.19}
\]

Substituting the above in eq. (2.15), the strain energy at a point on the vibrating free-free beam is then:

\[
U(x) = F \left[ \sum_{i=1}^{n} \beta_i^2 \{ \cosh (\beta_i x) - \cos (\beta_i x) - \sigma_i [\sinh (\beta_i x) - \sin (\beta_i x)] \} \right]^2 \tag{2.20}
\]

### 2.2.3 Vibration at a single mode

Plots for the theoretical strain energy associated with a beam vibrating at a single mode are presented in this section. The strain energy is calculated through the use of eq. (2.20) with \( n = i \), for the ith mode of interest. The theoretical modes, as calculated from eq. (2.17), have normalized amplitude. It is important to note that this means that the magnitude of strain energy associated with a given mode thus calculated will have no relevance, and it's only the relative magnitude along the beam that is significant.

**a) First mode.** Referring to Fig. 2.5, it is clearly evident that the strain energy is maximum at the mid-point of the beam. This is the point along the beam with maximum deflection for the first mode of vibration. It is important to realize that it is not the magnitude of deflection, but that of the square of its second derivative (which is radius of curvature as seen in eq. (2.12)) which causes the strain energy to be maximum at that point. This agrees well with the formulation of eqs. (2.11), (2.15) and (2.20).

**b) Second mode.** Fig. 2.6 further exemplifies the direct proportionality of the strain energy to the local radius of curvature. It is noteworthy that although the deflection magnitudes at the second mode are the opposite on the two sides of the mid-point, the strain energy density is the same. This is due to the fact that the strain energy is proportional to the square of the second derivative.
Figure 2.5 Theoretical deflection and strain energy for mode I
Figure 2.6  Theoretical deflection and strain energy for mode II
c) Higher modes. Figs. (2.7) - (2.9) give us a measure of how the strain energy variation along the beam starts getting complex as we look at the higher modes. This is in line with the complex mode shapes associated with higher modes. It can be seen that for the third mode, the strain energy peak in the middle is lower than the other two on either side.

As a final remark, then, the strain energy is low at points along the beam where the deflection appears flat, or where the radius of curvature is very high. This is very intuitive since when there is no deflection of the beam, the radius of curvature would be infinity, and eq. (2.11) would predict a strain energy of zero. This is expected, since no energy is required to produce a deflection of zero.

On the other hand, the strain energy is high at points along the beam where the deflection appears to make the beam circular in shape (local curvature), or where the radius of curvature is low. From eq. (2.11) then we do expect a high strain energy. This is also very intuitive since a large amount of energy is required to produce a highly curved beam. And as we get to higher modes of vibration, we need a higher energy to produce curvature at regular intervals on the beam for an equivalent deflection magnitude.

2.2.4 Vibration with multiple modes

When the beam vibrates under broadband forced excitation, a number of modes are excited at the same time. Recall that the total deflection of the beam, eq. 2.16, is a summation over the individual modes. Since all the modes have their characteristic shapes, it is readily seen that the summation of deflections over more than one mode will result in a complicated shape of total deflection. This in turn means that the regularity of regions of high strain energy seen in the last section should no longer be expected.

It is important to realize that in a real excitation the different modes will normally not vibrate in phase with each other. This implies that the individual modes will not reach their respective maximum amplitudes of deflection at exactly the same moment in time. We will confront this issue in the next section. For now we shall overlook the phase differences between the different modes, and simply look at the result of adding the deflections of the individual modes as seen in Fig. 2.5-2.7.
a) Deflection $Y_3(x)$

b) Strain energy $U_3(x)$

Figure 2.7 Theoretical deflection and strain energy for mode III
Figure 2.8 Theoretical deflection and strain energy for mode IV
Figure 2.9 Theoretical deflection and strain energy for mode V
Figure 2.10  Theoretical deflection and strain energy for modes I + II
As a simple example to visualize the effect of multiple modes, let us consider the case when the beam is vibrating at the first two natural frequencies (first two modes). Upon adding the individual mode deflections in Figs. 2.5(a) and 2.5(b), using eq. (2.16), the resultant deflection looks like Fig. 2.10(a). The strain energy associated with this composite deflection, obtained using eq. (2.20), looks like Fig. 2.10(b).

Before we interpret these plots, let us rewrite eqs. (2.16) and (2.20) for this particular case. Representing the composite displacement as \( Y_{12}(x) \), eq. (2.16) is:

\[
Y_{12}(x) = Y_1(x) + Y_2(x)
\]

and using eq. (2.15), the strain energy is:

\[
\varepsilon_{12}(x) = F \left[ Y_1^{''}(x) + (Y_2^{''}(x)) \right]^2
\]

or in expanded form:

\[
\varepsilon_{12}(x) = F \left[ (Y_1^{''}(x))^2 + (Y_2^{''}(x))^2 + 2 Y_1^{''}(x) Y_2^{''}(x) \right]
\]

Since \( \varepsilon_i(x) \propto (Y_i^{''}(x))^2 \), from eq. (2.15), we have:

\[
\varepsilon_{12}(x) = F \left[ \varepsilon_1(x) + \varepsilon_2(x) + 2 \sqrt{\varepsilon_1(x) \varepsilon_2(x)} \right]
\]

Since the modes are added linearly to obtain the composite deflection (eq. (2.21)), the total deflection at a particular point may either be the sum or the difference of the two individual deflections. The latter is possible since there are points along the beam where the modes may have deflections opposite to each other. This interference of the modes results in the complex composite deflection of Fig. 2.10(a).

From eq. (2.24) we can clearly see that the strain energy associated with this composite displacement is a function of the linear sum of the individual modal strain energies, plus an extra term. This extra term is twice the product of the second derivatives of the modal deflections. It is critical to note that this term can be negative (it is only its square, to which strain energy is proportional, that is non-negative). It is this very term that interferes with the sum of the individual strain energies, and produces the complex strain energy pattern of Fig. 2.10(b). These two terms are plotted in Fig. 2.11.
a) 1st term, $\psi_1(x) + \psi_2(x)$

b) 2nd term, $2 \sqrt{\psi_1(x) \psi_2(x)}$

Figure 2.11 The two terms in $\psi_{12}(x)$, eq. (2.24)
2.3 Time Varying Strain Energy

The strain energy we have been looking at is for static deflection of the beam in each mode of vibration. This would pertain to a snapshot of the beam during vibration, with the relative phases between modes neglected. When the beam is vibrating, though, each of the modes changes in amplitude over time. This variation can just be modeled as being sinusoidal [22], with appropriate frequencies of motion and relative phases taken into consideration while adding up the individual modes, to obtain the total deflection of the beam. We can thus replace the \( Y_i(x) \) in eq. (2.16) by both a time and position varying deflection \( Y_i(x, t) \). Using separation of variable approach, this can be written as:

\[
Y_i(x, t) = G_i(x) \ast H_i(t)
\]

(2.25)

The sinusoidal variation of \( H_i(t) \) is:

\[
H_i(t) = A_i \sin(\omega_i t + \phi_i)
\]

(2.26)

where

\[
G_i(x) = Y_i(x) \text{ in the section 2.2}
\]

\[
A_i = \text{amplitude of the } i \text{th mode of vibration}
\]

\[
\omega_i = \text{ith resonant frequency}
\]

\[
\phi_i = \text{phase of the } i \text{th mode relative to some basis}
\]

Substituting the above and eq. 2.17 into eq. (2.25), the time-varying deflection of each mode can be written as:

\[
Y_i(x, t) = A_i \sin(\omega_i t + \phi_i) \ast \{ \cosh(\beta_i x) + \cos(\beta_i x) - \sigma_i [\sinh(\beta_i x) + \sin(\beta_i x)] \}
\]

(2.27)

and the corresponding strain energy can then be written as:

\[
\psi(x, t) = A_i \sin(\omega_i t + \phi_i) \ast \beta_i^2 \{ \cosh(\beta_i x) - \cos(\beta_i x) - \sigma_i [\sinh(\beta_i x) - \sin(\beta_i x)] \}
\]

(2.28)
Figure 2.12 Real-time deflection and strain energy for mode I (for its 1 T)
Figure 2.13 Real-time deflection and strain energy for mode II (for 1 T of mode I)
Figure 2.14 Real-time deflection and strain energy for mode III (for T/2 of mode I)
2.3.1 Vibration at a single mode

A plot of the time-varying strain energy for the first three modes of the free-free beam is shown in Figs. 2.12 - 2.14. Note that the relative phases do not come into play here since only one mode each is present. Of particular interest is the complex variation of the deflection, and the corresponding strain energy with time and position. From elementary vibrations, though, we know that the only effect of variation with time is in the amplitude at a point. Thus, even though there is variation over time, the relative deflection and strain energy over position stays similar to that seen in Figs. 2.5-2.7.

2.3.2 Vibration at multiple modes

2.3.2.1 First two modes present

As stated before, for any formulation of vibration of the beam with multiple modes, we need to account for the relative phase information for each mode. We also need to consider the relative magnitudes of displacement between modes. This information can only be obtained from experimental data for the free-free beam. The frequency response function (FRF) data used for this section is plotted in Fig. (2.15). A chirp signal was used to excite the beam for this data. (A complete discussion of the test parameters used for this work is presented in Chapter 3.) The information pertinent to this discussion, and extracted from the FRF is tabulated in Table (2.1).

In Fig. 2.16, the effect of both the modes one and two being present simultaneously is shown, with the relative phases from experimental data being accounted for. It is important to note that since the second mode is at a higher natural frequency, there is more variation in equal time for the second mode than for the first. This is also evident from the individual modal plots of Fig. 2.12-2.13.

These two properties imply that we should expect a lot of complex variation of the strain energy over time. Indeed this is what is seen in Fig. (2.16).
Figure (2.15) Experimental FRF data for a free-free beam

Table 2.1 Experimental data for the free-free beam (First five modes)

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Frequency (Hz)</th>
<th>Amplitude (m/N)</th>
<th>Driving point Phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>37.35</td>
<td>2.30E-02</td>
<td>82.97</td>
</tr>
<tr>
<td>II</td>
<td>94.48</td>
<td>7.73E-03</td>
<td>-94.50</td>
</tr>
<tr>
<td>III</td>
<td>196.29</td>
<td>7.03E-04</td>
<td>96.29</td>
</tr>
<tr>
<td>IV</td>
<td>301.76</td>
<td>7.62E-05</td>
<td>78.10</td>
</tr>
<tr>
<td>V</td>
<td>473.14</td>
<td>1.90E-05</td>
<td>82.06</td>
</tr>
</tbody>
</table>

* For the strain energy plots, only the relative phase information is needed. Hence (say) all the phases are taken relative to the first mode, whose phase then is taken as 0.0.
Figure 2.15 Real-time deflection and strain energy for modes I + II (for 1T of mode I)
However, as we can see from the relative amplitudes of the modes in Table (2.1), the effect of higher modes is usually small since they cause very little variation in the resultant deflection and strain energy and their effect can be neglected for analytical modeling of experimental data.

2.3.2.2 First five modes present

Since we are finally interested in evaluating the strain energy with the first five modes being present, let us look at the time varying total displacement and strain energy along the beam. Following a procedure similar to that employed in the previous section, this information is presented in Fig. (2.16). We observe a behavior similar to that in Fig. (2.16), only with much more variation in the strain energy.

Comparing Figs. (2.16) and (2.17), we again note that the higher modes cause only small variations in both the displacement and the strain energy. This is another reason why we chose to work with only the first five resonant modes of the beam. The effect of modes above the fifth mode is negligible to both the plotted parameters. However it should be noted that for analysis of constrained-layer damping (CLD) for other purposes such as acoustic radiation, the effect of higher modes may be significant on the resultant sound radiation.

2.3.2.3 Time-averaged strain energy

The strain energy pattern in Fig. (2.17) is visibly very complex. Also the nature of strain energy is similar to that of Fig. (2.10) for two modes, with additional variation over time. Recall that this complex strain energy pattern arises from the non-linear term in eq. (2.24). Such a term will exist for each pair of modes in the five modes present. Eq. (2.24) can then be written as:

\[
\psi_{\text{total}}(x, t) = \mathbf{F} \left[ \psi_1(x, t) + \psi_2(x, t) + \psi_3(x, t) \right] \\
+ 2 \sqrt{\psi_1(x, t) \psi_2(x, t)} + 2 \sqrt{\psi_1(x, t) \psi_3(x, t)} + ... + 2 \sqrt{\psi_4(x, t) \psi_5(x, t)} 
\]

(2.29)
Figure 2.16 Real-time deflection and strain energy for modes I to V (for 1T of mode I)
It would indeed be very difficult to characterize such a complex formulation of the strain energy variation. Let us again look at the nature of the nature of one of the square rooted terms in eq. (2.29), plotted in Fig. (2.17). It appears in the plots that the terms are symmetric over time. This means that each crest is matched by a trough of equivalent size at a different time. If this is true then these crests and trough will cancel each other if the strain energy is averaged over time.

Fig. (2.18) takes a look at the above suggested time-averaged strain energy. It is indeed observed that over integer multiples of the time period of the first mode, the strain energy takes a simple looking form. Between integer multiples of the time period of the first mode, the strain energy periodically varies around the equilibrium position. It is evident that this variation gets smaller and smaller as we look at a larger chunk of time. The reason why this works best for integer multiples of time period of the first mode is because it has the lowest frequency, and hence the highest time period. Thus the higher modes go through a large number of their time periods during a single time period for the first mode.

Before we go ahead and use the time-averaged strain energy further, a note should be made of the assumptions involved in taking such a time average:
1. After the excitation signal is applied to the beam, the inherent damping characteristics of the metal (plus that of the CLD if applied) cause the vibrations to decay. This decay is different for each mode. It is assumed here that this effect of modal damping is negligible when the time average of the strain energy is taken.
2. Such a time-average of the strain energy would be characteristic of the particular experimental setup being studied. Variations due to a difference in the excitation signal are discussed in the next section.
3. As before the time-averaged strain energy should only be used to interpret the relative variation of the strain energy along the beam. Although absolute numbers are not difficult to obtain using eq. (2.14) all along instead of eq. (2.15), they are not relevant for this study.
Figure (2.17) One of the non-linear terms in Eq. (2.29), $2 \sqrt{u_1(x, t) u_2(x, t)}$

Figure (2.18) Total strain energy averaged over fractions of mode 1's T
2.4 Excitation Dependent Strain Energy

It was mentioned in the previous section that the phase information was taken from experimental data. It is important to note that the relative phase depends highly on the nature of excitation (and initial conditions if applicable).

For true broadband excitation, such as an excitation using an impact hammer or a broadband noise signal, all the modes start off in phase. They even have the same initial conditions of no deflection at all points along the beam.

On the other hand when using a swept sine signal (or a chirp signal, as described in the next chapter) for excitation, each mode is excited only when the signal reaches its particular natural frequency. This implies that some modes (typically the lower ones for a chirp signal) get a "head start", adding to the relative phase effects. Also note that for such a signal, the initial conditions for the higher modes will not be those of zero displacement, as is the case for broadband excitation. All these factors may contribute to complexity in the analytical modeling of beam vibration, and that to that of the related strain energy densities.

In Fig. (2.19) two different chirp excitations and the beam response are plotted. Note that the beam response is the curve which lags in time to the excitation. The resultant strain energy distribution for each of these excitations is plotted in Fig. (2.20). Strain energy is calculated using all the five modes of the beam. It is observed that the strain energy plots are evidently different as expected from the above discussion, and effect of the using different chirp signals is not negligible.

Let us again look at what the time-averaged strain energy looks like for the two different cases. As seen in Fig. (2.21), for the two different excitation signals this comes out to be almost the same.

Therefore, even though the strain energy distribution along a free-free beam under excitation depends upon various factors, we can nevertheless characterize it with the plots Fig. (2.18) or (2.21). This is the very distribution on which this work will be built upon, in the next few chapters.
a) Chirp signal time period = 0.25 secs

b) Chirp signal time period = 0.33 secs

Figure (2.19) Time-domain plots of two chirp excitations and the resulting beam response
a) $U_{\text{total}}(x,t)$ for 0.25 secs chirp

b) $U_{\text{total}}(x,t)$ for 0.25 secs chirp

Figure (2.20) Strain energy distribution for two different chirps
Figure (2.21) Time averaged strain energy for the two chirp signals
3. EXPERIMENTAL SETUP AND DESIGN

3.1 Outline of Setup

3.1.1 Overview

The test structure, setup, data acquisition and other parameters to be used for the experiments are introduced in this section. It is important to understand and choose the correct parameters for each test equipment in order to obtain reliable experimental data.

Fig. (3.1) shows a typical layout for the experimental setup used for acquiring the vibratory response of structures. In the figure some of the standard equipment used is shown in detail. A generic test item is shown with its motion measured by one transducer system (an accelerometer), and the input force measured by a second transducer system (a force transducer). In either case, these transducers often employ either the piezoelectric or the strain gage type of sensing element.

The electronic signals from these transducers are amplified electronically and analyzed. A frequency analyzer is commonly employed for analysis purposes. A computer is then used to further process and store the data for interpretation. A critical analysis of commonly used equipment, can be found in varied literature such as [23], [24].

Figure (3.1) General layout of a vibratory measurement system
3.1.2 Various elements used in the measurement chain

The major elements that were actually used in the experiments for this work, with brief specifics are detailed in this section. The complete test setup is shown in Fig. (3.2). It is worthwhile to mention that various elements used may well be replaced by other equipment or measurement procedures. References are hence provided to back up the often critical choices to be made in the initial stage of the experiments.

3.1.2.1 Test Structure

Since the current work focuses on application of constrained-layer damping (CLD) treatments to metallic beams, a 26.5"x1"x0.25" steel beam was used as the test structure. The beam was applied free-free boundary conditions. The free-free beam was chosen over other commonly used boundary conditions such as fixed-free (cantilever), pinned-pinned, etc., for the following reasons:
1. The free-free beam is known to have the most replicable boundary condition. Measured response of other beams such as with fixed boundary conditions is highly dependent on the rigidity of the boundary, which is hard to replicate [24].
2. Experimental natural frequencies of the beam can easily be measured to within 5% of their theoretical counterparts (see Appendix I for details).

3.1.2.2 Test structure support

In practice it is not feasible to provide a truly free support structure for the free-free beam, since the testpiece must be held in some way. However, it is easy to approximate such a scenario by supporting the testpiece with very soft 'springs' such as a fishing line or elastic bands. Fishing lines were chosen for this test setup to ensure a firm vertical support but provide complete horizontal flexibility. Note that this is because the excitation signal is applied in the horizontal plane (Fig. (3.2)). It should be made certain that these strings pass over the two nodal points of the first mode of vibration of the free-free beam, which is roughly 21% of the length of the beam from either side.

The entire test structure, along with the supports, should have a natural frequency well below the first mode of the free-free beam. The above-stated string attachment points are chosen in fact to ensure that the structure does not interfere with the vibratory response of the beam.

3.1.2.3 Shaker or vibration exciter

The shaker used was a Bruel & Kjaer, type 4809. This medium sized shaker can provide a force of about 10lbf, and is suitable for excitation of small testpieces. While a larger shaker may be a wastage of resources, a smaller one will almost certainly have insufficient force to excite the testpiece around and at its natural frequencies. This is because at resonance a very small excitation can produce large responses from the testpiece.
3.1.2.4 Stinger

It is necessary to connect the driving platform of the shaker to the structure, while incorporating the force transducer at the other end, attached to the testpiece. Although the shaker is capable of applying force in only one direction, the structure usually responds multidirectionally. Since for accurate a measurement procedure, the excitation provided by the shaker should be the only force applied to the testpiece, it is extremely important that the attachment between the shaker and testpiece be of the correct stiffness.

The above mentioned problem is overcome by using a stinger, which is a thin metallic rod. The stinger has the characteristics of being stiff in the direction of intended excitation (to transmit all the shaker force), and relatively flexible in the all the other directions. This is illustrated in Fig. 3.3. For instance if the stinger is too stiff in a direction transverse to the shaker excitation, it will produce moments opposite to the natural bending of the testpiece. This translates to introducing extraneous force input into the system. Moreover, these moments may also be picked up by a sensitive force transducer and be erroneously interpreted as axial input force.

The importance of correct stinger stiffness cannot be overemphasized. An unsuitable stinger can change the vibration response of the testpiece enormously. For
more information on how a correct stinger was chosen for the current work, and how a bad stinger affects system response, please refer to Appendix I.

### 3.1.2.5 Excitation signal fed by the shaker

We require a broadband excitation of the testpiece across the frequency spectrum in which the natural modes of vibration of the beam lie. There are several types of broadband excitation commonly used such as random broadband noise, pseudo-random noise, impact hammer, etc. While choosing the correct excitation signal is very important to get good experimental data, it is a wide ranging topic to be discussed here. Each type of excitation has its own disadvantages and advantages that depend heavily on the test objectives [24].

The particular type of broadband excitation used for this work is called a *chirp* signal. A chirp is a waveform whose instantaneous frequency increases linearly with time between two specified frequencies. The chirp is generated by selecting the lower $f_i$ and upper $f_u$ frequencies that act over the chirp’s time period $T_c$. The chirp is generated by the function:

$$ x(t) = A \sin[ 2\pi (f_i + \mu t/2) t + \alpha ] $$

where

- $A$ = amplitude
- $\alpha$ = arbitrary phase angle
- $\mu = (f_u - f_i) / T_c$

A typical chirp signal is plotted in Fig. (3.4). A critical analysis of the chirp signal and its comparison with other broadband excitation signals can be found in [25]. Let us now look at some of the major advantages that the chirp signal provides:

1. Concentration of power which maximizes the signal-to-noise ratio.
2. Low peak-to-rms amplitude ratio (crest factor).
3. Filter leakage can be reduced considerably by a proper selection of variables [24].
4. Greater control of both amplitude and frequency content of the input signal.
Figure (3.4) Chirp excitation signal, $f_i = 100$ Hz, $f_u = 400$ Hz, $T_c = 0.1$ secs
3.1.2.6 Transducers

The force transducer used to measure the input force signal was a PCB (Piezotronics), type 208B (S. No. 3157, sensitivity: 531.0 mV/lbf). The accelerometer used to measure the output acceleration signal was a PCB, type UA 353 A17 (S.No. 2517, sensitivity: 13.17 mV/g, mass 2.5 gm). Care should be taken that the transducers not be sensitive to base strains. Also the mass of the accelerometer should be negligible as compared to the mass of the testpiece, otherwise a shift in natural frequencies can be expected as the accelerometer is moved to different points along the beam.

The accelerometer was attached to points along the beam using wax. The force transducer was screwed to a thin metal strip with threads at one end, and a flat surface at the other end. This flat surface was attached to the beam using super glue. Care was taken to attach the force transducer to the same point of the beam between experiments.

3.1.2.7 Amplifiers and filters

A Bruel & Kjaer power amplifier, type 2706 was used to amplify the input chirp signal to the shaker. An Ithaco amplifier was used to amplify both the transducer signals before they were fed into the computer for data acquisition. This is a necessary step to ensure that the dynamic range of the data acquisition equipment be used favorably in order to get good resolution. A Krohn-Hite filter was also used to filter the signals above the Nyquist cutoff frequency.

3.1.2.8 Windowing

Even though the system response and the excitation signals will theoretically have exponential asymptotic decay, the data acquisition equipment can only sample a fixed time window. Various windowing functions such as Rectangular (Flat-top), Hanning, Hamming, Kaiser-Bessel windows are in contemporary use depending on the input signal type. A Rectangular window is the preferred function for analyzing stationary random signals [23, 24].
For the data presented in this work, the use of a Hanning window compromised detection of the chirp signal's frequency content at the low and high ends. Thus, although the Hanning window is commonly used with shaker measurements, the Rectangular window was used in this study.

### 3.1.2.9 Computer

The Masscomp computer from Concurrent Engineering Corporation (CEC) was used for both the signal generation and data acquisition. The data acquisition and analysis is briefly discussed in the next section.

### 3.1.3 Input Output signals and data analysis

As shown in Fig. (3.5), the response of the beam was measured at 10 equally spaced points along the length of the beam. The force transducer was attached to the beam behind measurement point #1. 10 measurement points are required in order to estimate the mode shapes for modes I through V. The accelerometer was moved from point to point between data acquisition routines.

The input signal measured by the force transducer translates to a force input
(Newtons or lbf) when the proper calibration is applied for the transducer. Similarly the output signal translates to acceleration (ms\(^2\)). The various computer programs then used to get the desired frequency response function (FRF) (in acceleration / force) are listed below with their respective functions:

1. *rimpulseunnsf* : This program was used for the data acquisition and frequency (Fourier) analysis of the input and output channel signals. This program outputs a binary file with the autospectra for both the channels, and their cross-spectrum.

2. *spcasciwnsf, spcasciwsam* : This program takes the above file and converts it to an ascii file, calculating either the FRF (either H1 or H2) or the coherence en route.

3. *asciispcw* : The above file is then transferred over the network to a directory where the analysis can be done and the data finally stored. This program extracts the data from this file and outputs it to another binary file.

4. *spcstargu* : This program extracts the final parameter being looked at, say the H1 FRF, and outputs it to an ascii file.

5. *damp2.exe, adamp18.exe* : This program plots out the FRF data from the above file and then calculates the resonant frequencies, their magnitudes, relative phases, and the damping associated with each mode.

6. *unwrap.m, phchange.exe* : These routines were used to unwrap the phase information obtained from the above data. A brief discussion on unwrapped vs. wrapped phase and the need for unwrapping the phase information in given in Appendix II.

7. *gu.junk.m* : This routine was used to plot the mode shapes from the data obtained above.

### 3.2 Constrained-Layer Damping (CLD) Treatments

#### 3.2.1 Overview

We now turn our attention towards the CLD treatments that were applied to the free-free beam in order to characterize their effect on the vibratory response of the beam. Recall from chapter 1 that our goal is to apply CLD patches to varied portions of the
beam. This section details information about the CLD treatments themselves and the attachment procedure for applying the CLD patches.

3.2.2 Material used for CLD patches

A CLD treatment patch consists of a viscoelastic layer and a metallic constraining layer. These two layers are applied to the beam in such a fashion that the viscoelastic layer is sandwiched between the beam and the constraining layer. Both these layers are very light in weight. Therefore the CLD patches do not add any significant amount of mass to the original beam.

3.2.2.1 Viscoelastic layer

The viscoelastic layer used for this work was a proprietary acrylic viscoelastic polymer from 3M®, family ISD 112, and type 2015X 1205. ISD 112 has damping properties that peak in the ambient temperature range (0°C to 60 °C, 32 °F to 190 °F). Type 1205 of this polymer is 5 mils (or 0.127 mm) thick, and comes in sheets with the polymer sticky on both sides. This allows it to bond tightly to both the constraining layer and the base metal. More information on the properties of this polymer can be found in [26].

3.2.2.2 Constraining layer

Recall from chapter 1 that the elastic modulus of the constraining layer should be high enough to cause the viscoelastic layer to undergo shear deformation during vibration of the base metal. The constraining layers recommended by 3M were too thin to be used with a base metal of thickness 1/4", the thickness of our beam. Three different constraining layers were tested with the above mentioned viscoelastic layer. The constraining layer then chosen was 0.006" steel shim stock. The major reasons for choosing this constraining layer are outlined in Appendix III.
3.2.3 Procedure for application of a CLD treatment

The effectiveness of the CLD on damping the vibration response of the base structure is highly dependent on its successful attachment to the base metal. Presence of any air bubbles between either two of the three sandwiched layer is detrimental to its effectiveness. The following procedure should be hence followed:

1. Cut the constraining layer into the desired size of the CLD patch. Care should be taken to avoid bent edges in the metal. Since 0.006” steel is thin enough, it can be easily cut with a paper cutter without damaging the edges.

2. Clean the constraining layer with a solvent such as acetone. It is necessary for the surface to be dry and free of any wax, grease, dirt, oil, scale or any other contaminates. Either of these, if present, will act as a barrier to good intimate contact, especially since ISD 112 is pressure sensitive.

3. Next unwrap the viscoelastic material, taking care not to touch the sticky side. Hold the metal constraining layer over the viscoelastic tape, and firmly lay one corner down. Press the metal layer down, working from the starting corner to the corner opposite it. Use pressure from fingers to squeeze air bubbles from the metal-viscoelastic interface.

4. Use a utility knife to trim the viscoelastic edge to meet the metal edge. Take care to use a sharp knife, since a blunt one can shear the viscoelastic layer underneath.

5. Work out any remaining air bubbles.

6. Mark the location on the base metal where the CLD patch is to be applied. Clean the base metal similar to that in step 2 above.

7. Unwrap the protective paper layer from the damping patch, and press one corner of the patch on to the base metal. Gradually lower the patch onto the base metal while continually applying pressure to the patch. A squeegee or even a wood piece will help maintain uniform pressure across the patch area.

If the CLD patch to be applied is a small one, then step 7 above will not work well since the constraining layer will thwart any efforts to apply pressure while attaching the
patch to the base metal. In such a circumstance, the following steps can replace some of the above:

1. Cut the constraining layer to at least 3" long. Attach the viscoelastic layer to a portion of this metal piece from one corner.
2. Then cut the extra viscoelastic layer off the constraining layer using a utility knife. Again make sure the knife has sharp edges. Make sure the protective layer on the viscoelastic material is left on during this procedure.
3. Mark the top side of the CLD patch with the desired length with a utility knife, making fairly visible grooves along the marked line.
4. Now attach the patch to the base metal as explained in the main steps above. The extra length of the constraining layer should now allow for bending when pressure is being applied during attachment.
5. After attaching the patch completely, shear the extra constraining layer portion off the patch, working on the groove made in step 3.

### 3.3 Experimental Design

#### 3.3.1 Overview

We now take a look at the experiments designed for this work. Our goal, again, is to characterize the effectiveness of damping patches applied to varied portions of the free-free beam. We know from chapter 1 that the effectiveness of CLD stems from the fact that there is dissipation of vibrational energy during bending of the beam, under vibration. In chapter 2 we looked at the complex variation of strain energy of the free-free beam along its length. However we made certain assumptions and we were able to reduce the variation in all cases to that in Figs. (2.16) and (2.19).

Now that we know what the variation of strain energy is, and that CLD dissipates this very energy during vibration of the beam under excitation, we can design our experiments to test certain hypotheses.
3.3.2 Basis for design of treatments

From the discussion at the end of chapter 2 concerning time-averaged strain energy, eq. (2.29) then simplifies to:

\[ <u_{\text{total}}(x)> = F \left[ u_1(x) + u_2(x) + u_3(x) + u_4(x) + u_5(x) \right] \]

(3.2)

where <> signifies a time averaged quantity.

Note the absence of the cross-product terms and the variation due to time. Also note that the relative magnitude of the individual strain energy terms depends on the relative magnitudes of the modal amplitudes, as pointed out earlier. These magnitudes are the numbers that we will require from the experimental data for each CLD treatment patch that we apply to the free-free beam.

3.3.3 Hypotheses for the experiments

We will assume that the when the CLD dissipates vibrational energy at a certain point along the beam, it does so from the total strain energy present at that point, \( u_{\text{total}}(x) \). This assumption is very plausible since all that CLD does is to dissipate energy, it does not care what this total energy is composed of, or which mode this energy comes from. Then if a fraction \( fr \) of the total strain energy is dissipated by CLD at a particular point:

\[ fr \times <u_{\text{total}}(x)> = F \left[ u_1(x) + u_2(x) + u_3(x) + u_4(x) + u_5(x) \right] \times fr \]

or further extending the hypothesis:

\[ fr \times <u_{\text{total}}(x)> = F \left[ fr \times u_1(x) + fr \times u_2(x) + fr \times u_3(x) + fr \times u_4(x) + fr \times u_5(x) \right] \]

(3.3)

This implies that if, say, the CLD patch dissipates 20% of the total energy at a particular point along the beam, it dissipates 20% from each modal strain energy. We
again take a look at the individual strain energy plots of Figs. (2.5) - (2.7). It is evident
that due to the characteristic strain energy distribution for each mode along the beam, the
total strain energy at a point may then be composed more of the strain energy for a
particular mode than another.

Also since lower modes have greater amplitude, they will have more strain energy
along the beam than the higher modes (recall that $\psi_i (x) \propto (Y_i (x))^2$). All the above
implies that even though we expect the same fraction of strain energy to be taken off each
mode, some modes will evidently lose more energy than others. Of course this itself
depends on the placement and the length of the CLD patch.

3.3.4 Treatments for the experiments

The treatments for this work are designed to work particularly at the third mode of
the free-free beam. Mode III was chosen because, one, it has a complex enough variation
along the length of the beam (Fig. (2.7)) which is absent in, say, mode I. Also higher
modes are typically very important from the point of view of acousticians, trying to reduce
the acoustically radiated noise from vibrating structures. Although we are only interested
in the effect of CLD on modal amplitudes of vibration, it is hoped such a foresight will
help in further development of concepts from this work.

In order to tailor the treatments for mode III, let us take a look at Fig. (2.7) again.
We observe three lobes of high strain energy along the length. If we want to target
reduction in the amplitude of mode III, we should apply the CLD treatments around these
lobes. The treatments patches thus chosen are shown in Figs. (3.6) - (3.9), and also
schematically drawn in Fig. (3.10) for clarity. The positions and the sizes of the patches
are also indicated in Fig. (3.10).

3.3.5 Nature of the treatments

Referring to Figs. (3.6) - (3.10) we can see that T1 is nothing but the bare beam
itself. This kind of a treatment is also called the control. T8 is the whole beam covered
with the CLD. This is the scenario in which CLD has been widely used in the past few
decades. It is called the sandwich beam. This treatment is considered so that we can compare the effectiveness of CLD patches versus sandwich treatments.

Each of the treatments T2-T7 covers 26% (an arbitrary number) of the strain energy for mode III of the free-free beam. Note that when these treatments are applied to the beam, the CLD patches will also cover strain energies of the other four modes. Note that Fig. (3.6) shows three treatments in the same plot while all the other plots show one treatment each. The following observations can readily be made from these plots:

1. T2 is the same as T4 except for the difference of covering the left lobe or the right lobe. These two lobes are mirror images of each other.

2. Since T3 covers the same percent strain energy for mode III under the patch, and the middle lobe of strain energy is lower than the side lobes, it has to be a longer patch.

3. T5-T7 are treatments with two patches each, but still covering the same percent strain energy for mode III as T2-T4. This means that each patch is now smaller than those in T2-T4. Each single patch covers 13% of strain energy for mode III.

4. T6 and T7 are identical in the manner that T2 and T4 are, as described above.

5. Following observation 2 above, we can readily see why the middle patches of T6 and T7 are wider than the side patches.
Figure (3.6) CLD treatment patches 2, 3, 4

Figure (3.7) CLD treatment patch 5
Figure (3.8) CLD treatment patch 6

Figure (3.9) CLD treatment patch 7
Figure (3.10) Schematic layout of all the treatments
CHAPTER 4. EXPERIMENTAL DATA & RESULTS

4.1 Reduction of Experimental Data

4.1.1 Introductory remarks

In this chapter the experimental data for the eight treatments, described at the end of the previous chapter, is presented. The experiments were conducted for a single beam and each treatment was replicated four times.

A Randomized Complete Block Design (RCBD) was followed for the experiments [27]. This means that four randomization’s were required to set up the sequence of the experiments. In each randomization (block) each of the eight treatments appeared exactly once. Therefore, in the sequence of the 32 (8x4) experiments, the (say) second replication of a treatment could be tested only if all the other treatments had been replicated at least once. As mentioned in [27], randomization is sort of an insurance against random occurrences, and helps against experimental bias.

The RCBD was chosen over the Completely Randomized Design (CRD) since the CRD would require to randomize all the 32 data sets. It was felt that having blocks (in RCBD) of complete sets of treatments dealt with reducing random error over time better than the CRD randomizations would do. This is because the experimenter is expected to improve (or worsen) in making precise measurements over time for a given setup.

While future work in this area will extend the datasets to more than one beam, just one beam was used in this work to make a preliminary investigation of the hypotheses before moving ahead with building a large database of experimental data.

The following symbolization is used in this chapter:

\[ T_x R_y \]

where

\[ T_x = \text{the } x\text{th treatment, } x = 1, 2, \ldots 8 \]

\[ R_y = \text{yth replication of the } x\text{th treatment, } y = 1, 2, 3, 4 \]
4.1.2 Choosing the correct FRF

As mentioned in chapter 3, the quantity measured from the experiments was the frequency response function (FRF), in units of acceleration per unit force. The FRF is given in either of the two methods:

\[
H_1 = \frac{G_{xy}}{G_{xx}} \tag{4.1}
\]

\[
H_2 = \frac{G_{yy}}{G_{yx}} \tag{4.2}
\]

where

\[H_1 = \text{FRF preferred when the signal to noise ratio of input is likely to be higher than that for the output}\]

\[H_2 = \text{FRF preferred when the signal to noise ratio of output is likely to be higher than that for the input}\]

\[G_{xx} = \text{Autospectrum of the force input}\]

\[G_{yy} = \text{Autospectrum of the beam acceleration output}\]

\[G_{xy} = \text{Cross-spectrum of input/output}\]

\[G_{yx} = G^*_{xy}\]

Another related quantity that measures the quality of experimental data is called coherence of the system. The coherence, \(\gamma^2_{xy}\), of the input-output system is defined as:

\[
\gamma^2_{xy} = \frac{H_1}{H_2} = \frac{|G_{xy}|^2}{G_{xx} G_{yy}} \tag{4.3}
\]

The coherence is always between [0,1], with 1 signifying a perfect signal with no noise, and 0 signifying an output signal totally uncorrelated to the input to the system. The true system FRF lies between the two measured FRF’s, and is equal to both \(H_1\) and \(H_2\) when the coherence is 1. Also, from eq. (4.3) we can see that the coherence is just the
ratio of the two FRF's. This means that H2 is always greater than H1 [23]. We will take a look at the system coherence for the setup for this work in section 4.1.2.4.

As mentioned in chapter 3, it is difficult for a shaker to put energy into the system near resonant frequencies of the testpiece. This means that the output signal to noise ratio is expected to be higher than that for the input, and H2 is expected to give more meaningful results [23]. Let us, however, take a look at both the H1 and H2 FRF's for a particular experiment and decide which quantity to use for our experimental data.

All the FRF plots shown in this chapter are for the measurement point 10 along the beam (chapter 3). This point was chosen because for at least one of the five modes any other point along the point is either a node point or lies close to one. Since the beam response for that mode would be almost zero at such a point, the FRF data would be close to the noise levels.

**4.1.2.1 Data sets where H2 is better**

Figs. (4.1) and (4.2) show H1 and H2 for the experimental data set R3T5. By looking at the complete frequency spectrum, we cannot make out much difference between the two quantities. The resonant frequencies are practically the same, but we can observe a little difference in the amplitude of the response for modes I and II of the beam, with H2 greater than H1 as expected.

However if we zoom into the peak for the first mode, as shown in Figs. (4.3) and (4.4), we can readily see a huge difference. The peak for H2 is well defined and agrees with theory, whereas that for H1 is unarguably affected by the bad signal to noise ratio at the first resonant frequency of the beam.

The reason for such a peak for in H1 is stated in section 4.1.2.4 This peak for H1 can obviously not be used for data analysis since it will certainly give errant magnitudes for the mode I response. However the following was observed for the 32 data sets:
1. H1 showed this behavior only for the first mode.
2. This behavior was observed in only 4 out of the 32 data sets. This comes out to be 12.5% of mode I peaks, and 2.5% of all the resonant peaks in the data sets (4/(32*5)).
Figure (4.1) H2 for R3T5

Figure (4.2) H1 for R3T5
Figure (4.3) H2 for R3T5, zoomed near mode I

Figure (4.4) H1 for R3T5, zoomed near mode I
4.1.2.2 Data sets where H1 is better

H2 FRF’s were not found to be better than H1 in all the data sets, as expected from theory. Let us compare the two for the data set R3T1, as shown in Figs. (4.5) and (4.6). Again, not much can be said about the difference between the two from the complete frequency spectrum. Even zooming the data around resonant peaks did not provide any visible difference other than the magnitude of response.

However, if we take a look at the mode shapes obtained by these two FRF’s, we can see that there is a huge difference in the measured response. In Figs. (4.7) - (4.10), the first two modes are plotted for both the response functions. The following observations can be easily made based on the theoretical mode shapes plotted in chapter 2:

1. Mode I shape is much closer to its theoretical counterpart for H1 than it is for H2.
2. Mode II shape for H2 does not correspond at all to the theoretical mode shape. On the other hand the mode shape for H2 agrees well with theory.

From the above observations, it can be easily concluded that the H2 data for such instances is clearly not acceptable for analysis. The following was observed to be true for the 32 data sets:

1. H2 showed this behavior only for modes I and II of the beam.
2. 17 out of 32 mode I shapes were found to deviate significantly from theory. (53% of mode I’s).
3. 5 out of 32 mode II shapes deviated significantly from theory. (16% of mode II’s)
4. Observations 2 and 3 above translate to 13.8% bad mode shapes for all modes in all the data sets. ((17+5)/(32*5)).

4.1.2.3 H1 provides more reliable information

From the previous two sections we can conclude that H1 will provide more reliable information for this work. All the data sets that visibly contain errant information will have to be discarded. This would mean more than 50% of all the data obtained using H2 as compared to 12.5% of data for H1’s. Clearly there is no competition.
Figure (4.5) H2 for R3T1

Figure (4.6) H1 for R3T1
Figure (4.7) Mode I from H2 for R3T1

Figure (4.8) Mode I from H1 for R3T1
Figure (4.9) Mode II from H2 for R3T1

Figure (4.10) Mode II from H1 for R3T1
4.1.2.4 Reasons for bad data sets

In sections 4.1.2.1 and 4.1.2.2, bad data was observed only for the first two modes of the beam. Modes III, IV and V had both good mode shapes, and good resonant peaks for both H1 and H2. This section tries to explain the plausible reasons for the above.

As mentioned before, we expect the input signal to noise ratio to be low at resonant frequencies of the beam. This translates to a low coherence at these frequencies since the system response would be based on a signal that does not have the amplitude a lot higher than what the noise is at that frequency. This is clearly observed in Fig. (4.11).

Note how the coherence dips exactly at the system resonances, especially for modes I and II. All the dips marked by an asterisk are due to the system response at integer multiples of the frequency at which electricity is supplied to the measurement devices, i.e at 60 Hz, 120 Hz, 180 Hz, etc. This is expected to have bad coherence since the system response has nothing to do with this electrical noise. We will neglect these dips from now on.

Also from eq. (4.3) we can see that the coherence is the ratio of the squared magnitude of the cross-spectrums divided by the two autospectrums. Let us take a look at each of these terms in order to understand the reasons for getting a bad coherence in the system. These three quantities are plotted in Figs. (4.12) - (4.14).

The most interesting of these quantities to observe is the autospectrum of the input, the $G_{xx}$, in Fig. (4.12). It clearly shows how the shaker has difficulty in putting energy into the system at the resonant frequencies of the beam, especially for modes I and II. In fact for mode I, the autospectrum at resonance is almost 100 dB below its peak across the spectrum. Most of the measurement systems will not be able to precisely measure across such a dynamic range.

This then explains why the H1, which is calculated using $G_{xx}$ as seen in eq. (4.1), does bad at the first two resonant peaks. On the other hand H2, as seen from eq. (4.2), uses $G_{yy}$, which does comparably a lot better than $G_{xx}$ at the resonant frequencies. This can easily be seen from Fig. (4.13). $G_{xy}$, which is used in calculating both these FRF’s is also well behaved, as seen from Fig. (4.14).
Figure (4.11) System coherence for R3T5

Figure (4.12) Input autospectrum, $G_{xx}$, for R3T5
Figure (4.13) Output autospectrum, $G_{yy}$, for R3T5

Figure (4.14) System cross-spectrum, $G_{yx}$, for R3T5
Since the coherence is so low at the first two resonant frequencies, as low as 0.1 in many cases, it seems H2 overestimates the response of the system a bit more than would be expected. This would be the only plausible explanation for getting bad mode shapes while using H2.

Also note that for modes III, IV, and V the system coherence is good. In fact it is almost 1. This explains why H1 and H2 agree well at these frequencies, and how resonant peaks and the mode shapes come out to be good by using either method.

4.2 Experimental Data

4.2.1 Introduction

As mentioned in section 4.1.2.2, 4 out of 32 data sets showed bad resonant peaks for mode I of the beam. These data sets were, therefore, rejected. These were namely R3T1, R2T2, R3T5 and R2T7. In the data presented and analyzed hereon, these data sets are excluded.

Since we are only interested in observing the change in vibration magnitudes for the five resonant modes of the beam, the data presented here will be in the form of histograms. It is important to note here the resonant frequencies of the beam shift slightly in frequency when the constrained-layer damping (CLD) treatments are applied. This is due to a change in the resultant stiffness of the beam. Again, this change is not the related to the purpose of this work, and will not be mentioned henceforth.

4.2.2 FRF data for all the treatments

In Figs. (4.14) and (4.15) the magnitudes of FRF, both from H1 and H2 are shown, in order to show that the replication obtained from H1 is better. We can easily observe that while the variation of H2 is around 7-10 dB for modes I and II, that for H1 is only 2-3 dB. Clearly, we have chosen the correct form of FRF to look at.

In the eight histograms presented in Figs. (4.15) - (4.22), the magnitudes of the FRF at the five resonant modes of the beam are presented.
Figure (4.15) FRF magnitudes for resonant modes, T1 from H2

Figure (4.16) FRF magnitudes for resonant modes, T1 from H1
Figure (4.17) FRF magnitudes for resonant modes, T2

Figure (4.18) FRF magnitudes for resonant modes, T3
Figure (4.19) FRF magnitudes for resonant modes, T4

Figure (4.20) FRF magnitudes for resonant modes, T5
Figure (4.21) FRF magnitudes for resonant modes, T6

Figure (4.22) FRF magnitudes for resonant modes, T7
These eight histograms correspond directly to the eight treatments in our data. For each mode all the replications are shown in order to observe the repeatability of the data sets. Also note that treatments T1, T2, T5 and T7 have only 3 replications due to the four data sets rejected, as mentioned previously.

The following initial observations about the nature of the data sets can be made:
1. The most variation of vibration magnitude is observed for modes I and II. This can be tied back to the low coherence at these modes.
2. As the magnitude of vibration decreases for modes I and II, the replication gets better in most of the cases. This can be explained by the fact that as the vibration amplitude decreases, the shaker is able to put in more energy into the system leading to a better signal to noise ratio and a better coherence.
3. Even for lower magnitudes of modes I, especially in T3 and T6, there is a lot of variation between replications. Since the replication for the control T1, is good, and the coherences for all cases in T3 and T6 were also found to be acceptable, this can only be explained by the nature of the treatments.

As it was mentioned in chapter 3, the effectiveness of the CLD patches is very dependent on intimate contact between the three layers present therein. Even though all precautions were taken during the application of the CLD patches, it is almost impossible to avoid microscopic bubbles in the interface between the viscoelastic layer and the metallic layers. These bubbles are detrimental to the effectiveness of the CLD in reducing the vibration response, and hence the variations observed in the vibration amplitudes for T3 and T6.

The reason why the same is not seen for the higher modes is probably because of two reasons. The first may be tied in to the near perfect coherences at these frequencies. The second reason is that the biggest contribution of the overall strain energy at a particular point along the beam is due to modes I and II. This is what we saw in chapter 2, due to strain energy being directly proportional to the square of the amplitude of vibration, and the fact that the higher modes have lower magnitudes of vibration.

4.2.3 A note on the mode shapes

Recall that all the above FRF plots are only for the measurement point 10 along the beam. But we are looking to quantify the change in magnitudes of vibration all along the beam. For this we should consider all the measurement points. However we would not have to worry about the other points if the CLD patches had a global effect on the beam. This would mean that each point along the beam would be reduced in vibration amplitude by an amount proportional to its amplitude in the untreated beam.

We can easily test this by looking at the mode shapes for all the five modes in all the eight treatment cases. This information is presented in Figs. (4.24) - (4.28). The following observations can be made from the mode shapes:

1. All the five mode shapes retain their shape across the treatments.
Figure (4.24) Mode I shapes for all the treatments, y-axis in m/N
Figure (4.25) Mode II shapes for all the treatments, y-axis in m/N
Figure (4.26) Mode III shapes for all the treatments, y-axis in m/N
Figure (4.27) Mode IV shapes for all the treatments, y-axis in m/N
Figure (4.28) Mode V shapes for all the treatments, y-axis in m/N
2. Note that mode shapes IV and V look strange because the 10 measurement points along the beam are not sufficient to display the complete complex variation for these mode shapes (chapter 2).

3. The only change in mode shapes across the treatments is a change in modal amplitude. Since the mode shape remains the same, the relative amplitudes between the measurement points stays the same across all the points.

4. Across all the treatments amplitude at point 10 is higher than that at point 1, while theoretically it should stay the same due to symmetry across the mid-point in a free-free beam. This can be explained by the fact that the stinger is attached behind point 1, and this constrains the free movement of the beam at that point. On the other hand, the beam is unconstrained in motion at point 10.

4.2.3 Averaged data for the treatments

In order to quantify the change in magnitudes of vibration for the five modes of the beam, the replications were averaged to obtain one number for each mode in each treatment. Tables (4.1) - (4.8) present this information.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>548.78</td>
</tr>
<tr>
<td>II</td>
<td>1286.90</td>
</tr>
<tr>
<td>III</td>
<td>347.72</td>
</tr>
<tr>
<td>IV</td>
<td>107.22</td>
</tr>
<tr>
<td>V</td>
<td>28.29</td>
</tr>
</tbody>
</table>
Table (4.2) Modal magnitudes and reductions for T2

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>498.59</td>
<td>0.8</td>
</tr>
<tr>
<td>II</td>
<td>298.35</td>
<td>12.7</td>
</tr>
<tr>
<td>III</td>
<td>112.72</td>
<td>9.8</td>
</tr>
<tr>
<td>IV</td>
<td>71.22</td>
<td>3.6</td>
</tr>
<tr>
<td>V</td>
<td>27.14</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table (4.3) Modal magnitudes and reductions for T3

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>306.20</td>
<td>5.1</td>
</tr>
<tr>
<td>II</td>
<td>1087.79</td>
<td>1.5</td>
</tr>
<tr>
<td>III</td>
<td>99.30</td>
<td>10.9</td>
</tr>
<tr>
<td>IV</td>
<td>81.23</td>
<td>2.4</td>
</tr>
<tr>
<td>V</td>
<td>20.68</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table (4.4) Modal magnitudes and reductions for T4

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>518.03</td>
<td>0.5</td>
</tr>
<tr>
<td>II</td>
<td>340.84</td>
<td>11.5</td>
</tr>
<tr>
<td>III</td>
<td>109.99</td>
<td>10.0</td>
</tr>
<tr>
<td>IV</td>
<td>66.33</td>
<td>4.2</td>
</tr>
<tr>
<td>V</td>
<td>26.75</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Table (4.5) Modal magnitudes and reductions for T5

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>397.09</td>
<td>2.8</td>
</tr>
<tr>
<td>II</td>
<td>667.4</td>
<td>5.7</td>
</tr>
<tr>
<td>III</td>
<td>195.58</td>
<td>5.0</td>
</tr>
<tr>
<td>IV</td>
<td>81.35</td>
<td>2.4</td>
</tr>
<tr>
<td>V</td>
<td>28.41</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table (4.6) Modal magnitudes and reductions for T6

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>419.88</td>
<td>2.3</td>
</tr>
<tr>
<td>II</td>
<td>1125.25</td>
<td>1.2</td>
</tr>
<tr>
<td>III</td>
<td>200.22</td>
<td>4.8</td>
</tr>
<tr>
<td>IV</td>
<td>100.96</td>
<td>0.5</td>
</tr>
<tr>
<td>V</td>
<td>27.72</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table (4.7) Modal magnitudes and reductions for T7

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>504.58</td>
<td>0.7</td>
</tr>
<tr>
<td>II</td>
<td>909.23</td>
<td>3.0</td>
</tr>
<tr>
<td>III</td>
<td>195.47</td>
<td>5.0</td>
</tr>
<tr>
<td>IV</td>
<td>94.59</td>
<td>1.1</td>
</tr>
<tr>
<td>V</td>
<td>26.52</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Table (4.8) Modal magnitudes and reductions for T8

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Averaged amplitude (acc/force)</th>
<th>Reduction from control (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>183.55</td>
<td>9.5</td>
</tr>
<tr>
<td>II</td>
<td>75.37</td>
<td>24.6</td>
</tr>
<tr>
<td>III</td>
<td>38.13</td>
<td>19.2</td>
</tr>
<tr>
<td>IV</td>
<td>25.77</td>
<td>12.4</td>
</tr>
<tr>
<td>V</td>
<td>9.84</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Table (4.9) Repeatability for the control, T1

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Repeatability (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>± 2.4</td>
</tr>
<tr>
<td>II</td>
<td>± 2.9</td>
</tr>
<tr>
<td>III</td>
<td>± 1.8</td>
</tr>
<tr>
<td>IV</td>
<td>± 0.5</td>
</tr>
<tr>
<td>V</td>
<td>± 0.2</td>
</tr>
</tbody>
</table>

Before any analysis can be made on the reduction of vibration amplitudes for the modes due to the CLD treatments, we must look at the repeatability of the data for the control. This is because all the reductions are to be measured against the control. The repeatability information is obtained by looking at the range in which a particular modal amplitude varied in the replications, and comparing it with the average for that mode. This data is presented in Table (4.9). Again the wide range of variation for modes I and II can be tied to the low coherence at their resonant peaks, as described in section 4.1.2.3.
4.3 Data Processing

4.3.1 Giving form to the hypothesis

The experimental hypothesis presented in chapter 3 is revisited to tie it with the experimental data. The main equation for the hypothesis, eq (3.3), states:

\[ \text{fr} \times <u_{\text{total}}(x)> = F \left[ \text{fr} \times u_1(x) + \text{fr} \times u_2(x) + \text{fr} \times u_3(x) + \text{fr} \times u_4(x) + \text{fr} \times u_5(x) \right] \]

(4.4)

The equation translates to saying that if the CLD patch takes off say, 20%, of the total vibrational energy at a point along the beam then it will take off 20% energy out of each of the five modes of the free-free beam. But since the contribution of each mode to the total strain energy at each point is different, this means that the effectiveness of the CLD patch will be proportional to this contribution.

4.3.2 Percentage of modal strain energy under patches

If we take a look at the CLD treatment patches presented in Fig. (3.10), in all the eight treatments there are only five types of patches. These patches, along with their symbols to be used henceforth are detailed below:

1. A side patch in a single patch treatment, for T2 and T4. \textit{(Patch 1)}
2. A middle patch in a single patch treatment, for T3. \textit{(Patch 2)}
3. A side patch in a double patch treatment, for side patches in T5, T6 and T7. \textit{(Patch 3)}
4. A middle patch in a double patch treatment, for T6 and T7. \textit{(Patch 4)}
5. A patch covering the whole beam, for T8. \textit{(Patch 5)}

In order to analyze the data, we must look at the percentage of the strain energy of each mode under the patch to the total strain energy for that mode across the length of the beam. This quantity, along with our hypothesis will enable us to calculate how effective a particular CLD patch is for each of the five modes of the resonant beam.

To calculate the above quantities, we will use the strain energy formulations developed in chapter 2, namely eq. (2.14). This states that:
\[ V_i(x) = F \left( \frac{d^2 Y_i(x)}{d x^2} \right)^2 \]  

(4.5)

where

\[ Y_i(x) = A_i Y_i'(x) \]

\[ A_i = \text{amplitude of vibration of the } i\text{th mode} \]

\[ Y_i'(x) = \text{normalized mode shape, with amplitude } = 1 \]

We will use the modal amplitudes from the data for the control, T1. We need to use the control data because this quantifies the untreated beam, and represents the magnitudes of vibration on which the CLD treatments act. These modal amplitudes are presented in Table (4.10). Using this information the strain energy information required for each mode for data analysis, as mentioned above is presented in Table (4.11).

The displacement for point 10 on the beam is scaled from the acceleration-force measurements from the FRF by taking into account the frequency squared correction. Also measurement point 10 is 0.75" away from one end of the free-free beam. Therefore, the displacement of point 10 is scaled (see Table (4.10) to obtain the maximum displacement amplitude of the modes based on the analytical mode shapes.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Resonant frequency (Hz)</th>
<th>Scaling* factor</th>
<th>Amplitude (m/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>35.07</td>
<td>0.867</td>
<td>1.425E-02</td>
</tr>
<tr>
<td>II</td>
<td>99.02</td>
<td>0.776</td>
<td>4.708E-03</td>
</tr>
<tr>
<td>III</td>
<td>187.40</td>
<td>0.711</td>
<td>3.883E-04</td>
</tr>
<tr>
<td>IV</td>
<td>333.80</td>
<td>0.709</td>
<td>3.786E-05</td>
</tr>
<tr>
<td>V</td>
<td>468.86</td>
<td>0.709</td>
<td>5.071E-06</td>
</tr>
</tbody>
</table>

* Scaling = analytical displacement of point 10 / analytical modal amplitude
Table (4.11) Percentage of modal strain energy under a treatment patch

<table>
<thead>
<tr>
<th>Patch Number</th>
<th>Strain energy of mode under patch / Total modal strain energy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode I</td>
</tr>
<tr>
<td>none*</td>
<td>54</td>
</tr>
<tr>
<td>1</td>
<td>6.2</td>
</tr>
<tr>
<td>2</td>
<td>45.3</td>
</tr>
<tr>
<td>3</td>
<td>2.6</td>
</tr>
<tr>
<td>4</td>
<td>17.8</td>
</tr>
<tr>
<td>5</td>
<td>100.0</td>
</tr>
</tbody>
</table>

* Percent strain energy contributions of each mode to the total strain energy across the beam

4.4 Data Analysis and Results

4.4.1 Introduction

After reducing, processing the experimental data and giving form to the hypothesis we are ready to test the hypothesis on the measured data. We will attempt to link the reductions in vibration amplitude of the modes, in Tables (4.1) - (4.8), to the percentage of modal strain energies presented in Table (4.11). This will be developed separately for different groups and sub-groups of CLD treatment patches.

Again note that our hypothesis predicts that the reduction in the vibration amplitude of a mode will be proportional to the percentage of its modal strain energy under the CLD patch. Also while looking at the reduction we will keep in mind the repeatability of the data, in Table (4.9). Reductions which lie in the error range will be referred to as insignificant reductions, while others will be referred to as significant reductions.
4.4.2 Single patch treatments

The treatments T2, T3 and T4 fall in this category. The different sub-groups of data for analysis follows. While explanation of actual reduction numbers starts with section 4.4.2.2, the next section talks about generic reductions.

4.4.2.1 Even and odd modes

If we look at the mode shapes for the different modes, as shown for the first three modes in chapter 2, we observe that odd modes have antinodes at the center of the beam while even modes have nodes at the beam. This means that there is a lot of strain energy associated with odd modes at the center of the beam, while little exists for the even modes. We should then expect to see reductions in amplitudes for odd modes with a center patch, as in T3 (Table (4.3)):

1. Modes I and III have significant reductions of 5.1 dB and 10.9 dB respectively, while mode II has almost no reduction (when compared to its repeatability).
2. Modes IV and V have complex mode shapes and hence a lot of variation in amplitude across the length of the beam (Figs. (2.8) and (2.9)). Also the strain energy for these modes is spread across the beam in four or five lobes. Also note that the center patch in T3 is about 5” long. This means that more than one lobe for mode V, and two partial lobes for mode IV will come under the patch.

Therefore the notion of even and odd modes in quantifying reductions does not apply well to modes IV and V, and they will be dealt with elsewhere in the following sections.

4.4.2.2 Mode I

For the single patch treatments, mode I shows reductions predicted by the hypothesis. Please refer to tables (4.1) - (4.4) and (4.11) for this section.

1. For both T2 and T4, the CLD patch covers only about 6.2% of the total strain energy for mode I in the beam. The patch is 3.5” long, covering 13% of the area of the beam. This is why we see no significant reductions in the mode I amplitudes for these treatments.
2. For T3, the CLD patch covers 45% of the total strain energy of mode I. This explains a reduction of 5.1 dB for mode I, which is a significant amount when compared to the maximum possible reduction in T8, 9.5 dB. Also, while the T3 patch is just 5" long the T8 covers the whole beam, 26.5".

4.4.2.3 Mode II

Again mode II shows reductions predicted by the hypothesis. Refer to tables (4.1) - (4.4) and (4.11) for this section.

1. For both T2 and T4, the CLD patch covers about 20% of the strain energy of mode II with patch length of 3.5" which is 13% of beam area. We would expect to see significant reductions then, and indeed we get 11-12 dB reductions for these treatments. Again this is a significant amount when compared to T8, where covering 100% of mode II strain energy achieves just double the reduction, 24 dB.

2. For T3, the CLD patch covers only 6.6% of the strain energy for mode II in the beam. This is why we see no significant reduction for mode II in this treatment.

4.4.2.4 Mode III

Recall from chapter 3 that the eight treatments were tailor made for mode III, with about 26% of the total mode III strain energy under each of the CLD patches. We would then expect that the reductions for mode III be the same across all the treatments. Again, refer to tables (4.1) - (4.4) and (4.11) for this section.

1. For each of T2, T3 and T4 a reduction of around 10 dB is obtained. This is exactly what was predicted by the hypothesis.

2. It is also important to note that we can obtain a reduction of 10 dB by covering just 26% of the strain energy of the beam (13% area), while with T8 and covering the whole beam (100% area and strain energy), we get a reduction just double, around 19 dB.

3. It is critical to note that even though the CLD patches in T2, T4 were 3.5" long, and that in T4 was 5" long, covering the same amount of strain energy of mode III provided the same reduction in its amplitude. This again proves that our hypothesis worked for
mode III in single patch treatments.

4.4.2.5 Modes IV and V

1. For T2 and T4, the CLD patch covers about 18% of the strain energy for mode IV, while it is 15% for T3. The reduction for T3 is 2.4 dB and that for T2, T4 is around 4 dB each.

2. Similarly while in T2 and T4 covering 11% of mode V strain energy does not provide significant reductions, covering 18% in T3 provides about 2.5 dB in reduction.

Even though the reductions are proportional to strain energy coverage, they are not as high as expected and seen in the cases for modes I-III. Turn to Table (4.10), and recall that \( \frac{1}{\alpha} (x) \alpha (Y_1 (x))^2 \). If we calculate the total strain energy for the beam at any point along the beam under the patches, the contributions of mode IV and V are on the order of 0.01-0.04%. It seems that since these numbers are almost negligible in comparison to contribution of the first three modes. A plausible explanation for low reductions for modes IV and V then would be that this contribution seems to be in the noise levels of strain energy contributions and are not seen by the CLD patches.

4.4.3 Double patch treatments

Treatments T5, T6 and T7 fall in this category. Although reductions in vibration for these treatments are found to be proportional to the amount of strain energy under the patch for a given mode, the magnitude of reductions is consistently lower than that of the single patch treatments. Again refer to tables (4.1) - (4.4) and (4.11). Also note that the strain energy coverage for a double patch will be the sum of the coverage for each individual patch.

Mode III: The percent strain energy covered by the patches is 26% in these three treatment cases. The reductions in all three treatments is consistent around 5 dB. The fact that these are consistent is predicted, but this is half of the reductions for similar coverage for single patches, at 10 dB.

Mode II: A middle patch present in both T6 and T7 covers just 0.4 % of its strain energy.
We expect this to not provide any significant reduction. The second patch in such a case covers 9% of the strain energy. These treatments provide little or no significant reductions, around 2-3 dB.

In T5 two such side patches, thus covering about 18% of the mode’s strain energy, provide around 5.5 dB of reduction. This is again half of reductions for similar coverage in single patches which give around 10 dB reduction for T2, T4.

**Mode I**: T5, T6 or T7 provide little or no significant reductions. The strain energy coverage for T6 and T7 is around 22%, while that for T5 is about 5%. While getting no reductions for T5 is predicted, low reductions for T6 and T7 again deviate from theory.

**Modes IV, V**: None of the double patches provide any significant reductions. This can again be explained by low energy contributions of modes IV and V to the pool of total energy, as for the single patch treatments.

### 4.4.4 CLD effectiveness as a function of strain energy covered

In the preceding sections we have observed that the CLD effectiveness for small patches compares well with its effectiveness for the case where the whole beam is covered, T8. Let us look at this effectiveness as a function of both the percent strain energy covered for a mode versus the ratio of the patch effectiveness to maximum possible reduction (T8). This relation is plotted in Fig. (4.23).

It is evident from this plot that to achieve any significant reduction in vibration amplitudes a nominal amount of percentage of the strain energy of a mode needs to be covered. However, by covering 20-45% of the strain energy of a mode (dependent on the mode we are looking at) we can achieve half the maximum possible reduction. This is even more significant in the light that all the CLD patches were under 5” in length, whereas to obtain maximum possible reduction we need to cover the whole beam, i.e 26.5".
Figure (4.23) Ratio of patch versus maximum possible effectiveness

4.5 Sources of error

In this section the major sources of error for the experimental data are listed:

1. For the control, the biggest source of error is the low signal to noise ratio for the input, quantified by the low coherence. The response of the beam shows variation especially at the resonant peaks.

2. For the treatments, minuscule bubbles between the CLD layers causes its effectiveness to vary a lot. This is because intimate contact between layers is critical for CLD effectiveness.

3. Peaks such as those shown in Fig. (4.24) were present in some of the data sets. This is definitely a problem of resolution. Even though 16384 data points were used in the frequency range 0-520 Hz, since the lower modes have low damping the peaks are very sharp and hence an even finer resolution would be required. Note that even though such
peaks may not affect the magnitude of vibration much, it affects the quality factor enormously. This is the reason why the quality factor was not used to quantify the effectiveness of the CLD treatments.

4. Even though all experiments were done very carefully, small shifts in the point of attachment of the stinger to the beam may be a possible source of error.

5. Lastly, as in all experiments, human error cannot be counted out. This is the reason why randomization was done, to reduce bias towards any particular treatment.
Figure (4.24) Two examples of slightly imperfect peaks, a source of error
CHAPTER 5. CONCLUSIONS

The following conclusions can be made from the experimental hypotheses and the measured data for this work:

1. The time averaged strain energy formulations proposed in chapter 2 agree well with the experimental data. This means that in order to assess the total strain energy in the beam available for dissipation to the CLD treatments can be done by taking just the sum of the modal strain energies.

   The cross product terms between all pairs of modal strain energies can then be neglected, and hence we can get rid of the non-linear behavior of time-varying strain energy. This also means that the strain energy in the beam is not dependent on the type of excitation for all practical purposes.

2. The experimental hypothesis that the effectiveness of a CLD patch is only dependent on the percentage of modal strain energy under the patch, holds well separately for the single and double patch treatments. Evenmore, this effectiveness is directly proportional to the amount of strain energy under the patch.

3. The above conclusion leads to another one. Since CLD effectiveness is only dependent on the percentage of energy under the patch, placement of the patch is not important as long as this percentage is maintained. Again this is separately true for single and double patch treatments.

4. Small patches of CLD treatments are very effective when compared to the maximum possible reductions, obtained by covering the entire beam. This optimized use of CLD is very important for cost-effective real world applications.
5. Mode picking property of the CLD patches is also evident from the results since the treatment’s were tailor made for mode III of the beam. Again this is separately true for single and double patch treatments.

6. Single patch treatments were found to be more effective than double patch treatments for the same percentage of modal strain energy targeted. In fact for most of the cases they gave twice as much dB reduction in the vibrational amplitudes.

There seem to be two plausible explanations for this phenomenon. One, literature provides studies on the optimized length of a patch for a particular frequencies. This length is derived by making note of the wavelength of the flexural waves in the CLD layers and then optimizing the length to obtain maximum dissipation in these layers [28]. Very small patches may be in conflict with such theory for most of the modes.

Another possibility is interactions between the two patches to have in effect a complex variation on the vibration amplitudes of the modes.

7. There seems to be a nominal strain energy amount above which the amount for a particular mode must be under a patch, for the CLD to be effective. This is based on the fact that small reductions were obtained for modes IV and V of the beam, which had a minuscule percentage contribution to the total strain energy in the beam.

**Recommendations for Future Work**

Based on the above conclusions, the following direction is recommended for future work:

1. More kinds of treatments of CLD patches should be tried in order to further validate the hypotheses presented in this work. These patches should involve targeting some of the other modes, especially higher ones since these are important to control acoustically radiated noise from structures.
2. The data analysis for the double patch treatments was largely inconclusive. The way to tackle multiple patches may be to first try and model individual patches, and then question if a superposition principle applies to the reductions in modal amplitudes.

3. Since the resonant peaks were not found to be perfect in many cases, analysis using damping factors of the beam could not be done. Such an analysis would require a better frequency resolution.

Since a very fine resolution was used for this work itself, the following is recommended. A new type of signal is proposed which would concentrate the data points around the resonant peaks, since fine resolution away from the peaks turns out to be a waste of resources. A chirp signal, for instance, could be made to behave in such a way easily using customizable data acquisition programs available in the university.

4. Sensitivity of the experiments to small shifts in stinger attachment position, fishing line position (used for suspending the free-free beam) should be studied. These will help in improving the repeatability of the experiments.

5. The hypotheses presented here should be applied to other types of structures, such as beams and plates with varied boundary conditions such as cantilever, simply-supported, etc.

6. A larger dataset of treatment patches applied on a number of similar beams should be developed. This will allow us to do statistical analysis of the data, which was not done in this preliminary investigation of the hypotheses presented in this work.
APPENDIX I: STINGER

A stinger is used to connect the driving platform of the shaker to the structure, usually incorporating a force transducer at the interface with the structure. The stinger should be designed to avoid adding any extraneous forces/moments into the system. The only force input to the measurement test structure should be the excitation signal fed through the shaker, applied normal to the plane of the structure. Therefore a stinger should have the following properties:
1. Very stiff in the direction of the force.
2. Flexible in all the other directions

For the test apparatus setup in this work, detailed in chapter 3, the stinger will affect the free-free beam response if it is too stiff in the directions transverse to that of the force input from the stinger. Since a bad stinger will add stiffness to the system by resisting bending moments in this transverse direction, the affect of such a stinger is observed in the free-free beam response by raising of the natural frequencies of the beam. This behavior will increase with the increase in stinger stiffness.

The procedure to choose an adequate stinger is then to match the theoretical frequencies of the beam as closely as possible with their experimental counterparts. The natural frequencies of a free-free beam are given as:

\[ \omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}} \]  \hspace{1cm} (AI.1)

where

\[ \omega_n = \text{nth natural frequency} \]
\[ \beta_n = \text{constant dependent on the length and free-free boundary conditions [22]} \]
\[ E = \text{elastic modulus of the material of the beam (estimated)} \]
\[ \rho = \text{density of the beam (estimated)} \]
\[ A = \text{cross-sectional area of the beam (measured)} \]
Table (A1.1) Theoretical natural frequencies of the free-free beam

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>36.4</td>
</tr>
<tr>
<td>II</td>
<td>100.3</td>
</tr>
<tr>
<td>III</td>
<td>196.7</td>
</tr>
<tr>
<td>IV</td>
<td>325.1</td>
</tr>
<tr>
<td>V</td>
<td>485.7</td>
</tr>
</tbody>
</table>

By substituting the values for the test beam used in this work, we can easily obtain the theoretical natural frequencies of the beam. These are tabulated in Table (A1.1). Note that the estimated and measured quantities above, and hence the calculated frequencies will have some error associated with them.

It can easily be shown that the stiffness of a stinger is directly proportional to its length. Therefore, the procedure to choose an appropriate stinger is to experiment with different lengths of the stinger and choose the one which provides the least stiffness. This would be the one which elicits a system response with natural frequencies as close to the theoretical ones as possible. A note is to be made here that a stinger can be too flexible and cause other complexities [23].

In the pages that follow, nine different stinger lengths were experimented with, within the range 5.8" - 11". The frequency response function (FRF) for point 10 along the beam (chapter 3), was chosen to make the measurements. FRF’s for systems with these different stinger lengths are plotted in Figs. (A1.1) - (A1.9).

The purpose of this Appendix is only to illustrate how a stinger can affect the system response and how an appropriate stinger is to be chosen. Therefore in the results that follow, no attempt is made to analyze the system response changes in the cases of different stinger lengths. This may be appropriate for another in-depth work in the area. It is then sufficient to graphically analyze these plots:
1. The stinger modes (marked by an asterisk in the plots) shift to higher frequencies as the stinger length is shortened.

2. When a stinger mode is very close to a natural frequency of the beam, both these modes shift from their expected frequencies. In this case the amplitude of the stinger mode increases at cost of the system mode near it.

As an aside it is also interesting to note that as the stinger resonance increases, the resonant frequency of the mode just above it shifts upward. This is easily seen for beam’s mode III when Figs. (AI.1) and (AI.2) are compared.

When the stinger resonance rises just above the mode III frequency, the mode III frequency shifts below it suddenly, as seen in Figs. (AI.2) and (AI.3). Then as the stinger frequency rises further, the mode II frequency shifts back to its original position, as seen when comparing Fig. (AI.4) and (AI.1).

A similar pattern is observed when the stinger comes close to, and passes above the mode IV of the beam. This is seen in Figs. (AI.5) - (AI.8).

Table (AI.2) tabulates the experimental natural frequencies of the beam that were found to be the closest to their theoretical counterparts in Table (AI.1). Note that the percentage error is very small. This is characteristic of using a free-free beam with a well designed stinger.
Figure (AI.1) FRF with stinger length = 11"

Figure (AI.2) FRF with stinger length = 10"
Figure (A1.3) FRF with chosen stinger, length = 9"

Figure (A1.4) FRF with stinger length = 8.8"
Figure (A1.5) FRF with stinger length = 8.3"

Figure (A1.6) FRF with stinger length = 7.9"
Figure (A1.7) FRF with stinger length = 7.3"

Figure (A1.8) FRF with stinger length = 6.7"
Figure (AI.9) FRF with stinger length = 5.8"

Table (AI.2) Best experimental frequencies, stinger length = 9.0"

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Experimental Frequency (Hz)</th>
<th>Deviation from theoretical (% error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>35.1</td>
<td>3.57</td>
</tr>
<tr>
<td>II</td>
<td>99.1</td>
<td>1.19</td>
</tr>
<tr>
<td>III</td>
<td>186.5</td>
<td>5.18</td>
</tr>
<tr>
<td>IV</td>
<td>331.0</td>
<td>1.81</td>
</tr>
<tr>
<td>V</td>
<td>469.3</td>
<td>3.38</td>
</tr>
</tbody>
</table>
APPENDIX II: UNWRAPPED PHASE

The system response measured for this work was the frequency response function (FRF) as mentioned in chapter 3. The FRF is given in either of the two methods:

\[
H_1 = \frac{G_{xy}}{G_{xx}} \quad \text{(AII.1)}
\]

\[
H_2 = \frac{G_{yx}}{G_{yy}} \quad \text{(AII.2)}
\]

where

\[H_1 = \text{FRF preferred when the signal to noise ratio of input is likely to be higher than that for the output}\]

\[H_2 = \text{FRF preferred when the signal to noise ratio of output is likely to be higher than that for the input}\]

\[G_{xx} = \text{Autospectrum of the force input}\]

\[G_{yy} = \text{Autospectrum of the beam acceleration output}\]

\[G_{xy} = \text{Cross-spectrum of input/output}\]

One or both of the above FRF's can be used to measure the system response depending on the system conditions, noise, etc. [24]. In either of the cases the FRF is a complex entity. The FRF, designated as \(H\), can be represented as:

\[
H = |H| e^{j\phi} \quad \text{(AII.3)}
\]

\[
\tan \phi = \text{Im}(H) / \text{Re}(H) \quad \text{(AII.4)}
\]

Naturally when a data acquisition system calculates the phase, \(\phi\), of the FRF it is clamped in the range \([-180^\circ, +180^\circ]\). The phase near resonance undergoes a 180° change [22], with the theoretical phase at resonance being 90°. A typical phase-frequency plot for a FRF of a free-free beam looks like Fig. (AII.1). Note the oscillation of the phase around the resonant frequencies. This is due to a rapid change of the phase near resonance, where
Figure (AII.1) Wrapped phase data for measurement point 3 of the beam

Figure (AII.1) Unwrapped phase data for measurement point 3 of the beam
the amplitude of vibration at that natural frequency tends to be very high and the signal to noise ratio of the output force signal is low.

The mode shape for a natural frequency of the free-free beam is obtained by plotting the amplitude of vibration at ten points along the beam (chapter 3), with the appropriate phases taken into account. These phases are taken relative to a certain point, usually taken to be the driving point of the system (point 1 for this work). For plotting out a mode shape it is sufficient to know whether each measurement point is either in or out of phase ($180^\circ$ phase difference). However if the phase information is taken from a plot such as Fig. (AII.1), these relative phases are more often than not, incorrect information. As mentioned before this is due to phase oscillation near resonance. This phase, called the wrapped phase, is unable to keep track whether the phase after the oscillation spree is $180^\circ$ out of phase or in phase with the phase before the oscillations began. This results in ugly mode shapes as shown in Figs. (AII.3) and (AII.4).

From the nature of the tangent function it is easy to see that a phase of $180^\circ$ is equal to a phase of $-180^\circ$. To make use of this fact, get rid of the ugly oscillations and resolve the phase ambiguity mentioned above we can look at the unwrapped phase data. This phase information can be easily obtained by the following process. A phase of $-179^\circ$ is interpreted as $+181^\circ$ degrees and vice versa. This gets rid of the nasty oscillations near $180^\circ$ and we get the unwrapped phase as plotted in Fig. (AII.2).

The intent of this Appendix is only to illustrate how the unwrapped phase can help in obtaining meaningful mode shapes. For this reason we leave with a graphical view of the five mode shapes obtained for the free-free beam from both the unwrapped and wrapped phase. These are plotted in Figs. (AII.5) - (AII.9).

The following observations are to be made from these plots:
1. The amplitude of vibration at a measurement point at a particular frequency stays the same with both the phase data.
2. The only change the unwrapped phase can make is to determine whether the amplitude should be negative or positive relative to the driving point, point 1.
Figure (AII.3) Bad mode I with wrapped phase data

Figure (AII.4) Bad mode II with wrapped phase data
Figure (AII.5) Mode I, --- with wrapped phase, --- with unwrapped phase

Figure (AII.6) Mode II, --- with wrapped phase, --- with unwrapped phase
Figure (AII.7) Mode III, — with wrapped phase, —— with unwrapped phase

Figure (AII.8) Mode IV, —— with wrapped phase, —— with unwrapped phase
Figure (AII.9) Mode V, -- with wrapped phase, --- with unwrapped phase
APPENDIX III: CONSTRAINING LAYER

The effectiveness of the constrained-layer damping (CLD) treatment in reducing the vibration response is highly dependent on the Elastic modulus (E) of the constraining layer. With the correct elastic moduli of the constraining layer and the base metal, the sandwiched viscoelastic layer undergoes shear deformation thereby dissipating energy. If the correct constraining layer is chosen, this dissipation of energy can be maximized.

The effectiveness of the CLD treatment is also dependent on intimate contact between the three layers involved. This is especially true of 3M's ISD 112, the viscoelastic layer used in this work, which is pressure sensitive. Therefore, the constraining layer chosen should such that it ensures a good bond to the viscoelastic layer.

The elastic modulus of the constraining layer is dependent on both its material, and its thickness. For this discussion, three constraining layers were chosen to be tested. The material to be chosen would be the one which would provide the most dissipation of energy in the viscoelastic layer, and which would ensure a good bond of the treatment to the free-free beam. The three constraining layers to be tested were:

1. Aluminum sheet, 0.012” thick.
2. Steel shim stock, 0.006” thick.
3. Steel shim stock, 0.002” thick.

One half side of the free-free beam was applied with CLD treatments using each of the three constraining layers. The same viscoelastic layer was used in all the three treatments. The system response for measurement point 10 (see chapter 3) for the untreated case and the three treatments is shown in Figs. (AIII.1) - (AIII.4).

A comparison of the amplitudes of the five modes is shown in Table (AIII.1). The following observations can be made:

1. Major differences in the system response were obtained for the first three modes.
2. 0.002” steel provided the least reduction in vibration amplitudes. The other two constraining layers provided roughly the same reduction.
Figure (AIII.1) FRF for untreated beam

Figure (AIII.2) FRF for beam with 0.002" steel constraining layer in treatment
Figure (AIII.3) FRF for beam with 0.006” steel constraining layer in treatment

Figure (AIII.4) FRF for beam with 0.012” Aluminum constraining layer in treatment
Table (AIII.1) Modal response amplitudes for the three treatments

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Response amplitude (acceleration/force)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No TRT*</td>
</tr>
<tr>
<td>I</td>
<td>661.6</td>
</tr>
<tr>
<td>II</td>
<td>298.9</td>
</tr>
<tr>
<td>III</td>
<td>628.4</td>
</tr>
<tr>
<td>IV</td>
<td>76.9</td>
</tr>
<tr>
<td>V</td>
<td>42.8</td>
</tr>
</tbody>
</table>

* TRT = Treatment

0.006” Steel constraining layer was much easier to apply than the 0.012” Aluminum layer. This is because it is difficult to apply pressure while attaching the CLD treatment to the beam if the constraining layer is thick. As mentioned in the CLD application procedure in chapter 3, it is very essential to apply pressure during attachment of the CLD treatment to the beam in order to ensure intimate contact.

Also, while 0.006” Steel can easily be cut using a paper cutter, 0.012” Aluminum can only be cut using heavy machinery. While tin clips may be used for the latter metal, this will produce bent edges during cutting. This renders the metal unsuitable to ensure a good bond of the CLD to the base metal.

Due to these reasons it was decided to use 0.006” Steel shim stock as the constraining layer for this work.
REFERENCES


